JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

VECTOR

(PRACTICE SHEET)

LEVEL- 1

Question based on **Kinds of vectors**

Q.1 If $\vec{a}$ is a constant vector then -

(A) the direction of $\vec{a}$ is constant

(B) the magnitude of $\vec{a}$ is constant

(C) both direction and magnitude of $\vec{a}$ is constant

(D) None of these

Q.2 If $\vec{a} = \vec{b}$, then

(A) both have equal magnitude and collinear

(B) both have equal magnitude and like vectors

(C) both have equal magnitude

(D) they have unequal magnitude but like vectors

Q.3 Two vectors will be equal when-

(A) they have same magnitude

(B) they have same direction

(C) they meet at a point

(D) their magnitude and direction is same

Q.4 Which of the following is unit vectors-

(A) $\hat{i}+\hat{j}$ (B) $\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{2}}$

(C) $\hat{i}+\hat{j}+\hat{k}$ (D) $\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

Q.5 Unit vector in the direction of $\vec{a}$ is represented by

(A) $1.\ \vec{a}$ (B) $\frac{\vec{a}}{|\vec{a}|}$

(C) $\vec{a}\,|\vec{a}|$ (D) $\frac{\vec{a}}{\hat{i}}$

Q.6 The zero vector has-

(A) no direction

(B) direction towards a particular point

(C) direction towards the origin

(D) indeterminate direction

Question based on **Addition & subtraction of vectors**

Q.7 If ABCDE is a pentagon, then $\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}$ equals-

(A) 3 $\overrightarrow{AD}$ (B) 3 $\overrightarrow{AC}$

(C) 3 $\overrightarrow{BE}$ (D) 3 $\overrightarrow{CE}$

Q.8 If $\vec{a} = 2\hat{i}+5\hat{j}$ and $\vec{b} = 2\hat{i}-\hat{j}$, then unit vector in the direction of $\vec{a}+\vec{b}$ is-

(A) $\hat{i}+\hat{j}$ (B) $\sqrt{2}\,(\hat{i}+\hat{j})$

(C) $(\hat{i}+\hat{j})/\sqrt{2}$ (D) $(\hat{i}-\hat{j})/\sqrt{2}$

Q.9 If $\vec{a}$ and $\vec{b}$ are two unit vectors then vector $(\vec{a}+\vec{b})$

(A) is a unit vector

(B) is not a unit vector

(C) can be a unit vector or not

(D) is a unit vector when both $\vec{a}$ and $\vec{b}$ are parallel

Q.10 If $\vec{a}$ and $\vec{b}$ represent vectors of two adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of a regular hexagon ABCDEF, then $\overrightarrow{AE}$ equals-

(A) $\vec{a}+\vec{b}$ (B) $\vec{a}-\vec{b}$

(C) $2\vec{b}$ (D) $2\vec{b}-\vec{a}$

Q.11 If in a parallelogram PQRS, sides PQ and QR are represented by vector $\vec{a}$ and $\vec{b}$ respectively then the side represented by $\vec{a}+\vec{b}$ is -

(A) $\overrightarrow{PR}$ (B) $\overrightarrow{RS}$ (C) $\overrightarrow{QS}$ (D) $\overrightarrow{PQ}$

Q.12 If ABCD is a quadrilateral, then the resultant of the forces represented by $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ is

(A) $2\overrightarrow{BA}$ (B) $2\overrightarrow{AC}$

(C) $2\overrightarrow{AD}$ (D) $2\overrightarrow{AB}$

Q.13 If ABCD is a rhombus whose diagonals cut at the origin O, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}$ =

(A) $\overrightarrow{AB}+\overrightarrow{AC}$ (B) $\overrightarrow{AB}+\overrightarrow{BC}$

(C) $2(\overrightarrow{AC}+\overrightarrow{BD})$ (D) $\vec{0}$

Q.14 If vector $\vec{a}$, $\vec{b}$ represent two consecutive sides of regular hexagon then the vectors representing remaining four sides in sequence are-

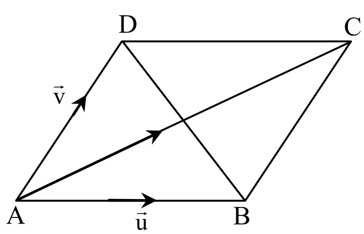
(A) $\vec{a}-\vec{b}$, $\vec{a}-\vec{b}$, $\vec{a}+\vec{b}$, $\vec{a}+\vec{b}$

(B) $\vec{a}-\vec{b}$, $\vec{a}$, $\vec{b}-\vec{a}$, $\vec{b}$

(C) $\vec{a}+\vec{b}$, $-\vec{a}$, $-\vec{b}$, $\vec{a}-\vec{b}$

(D) $\vec{b}-\vec{a}$, $-\vec{a}$, $-\vec{b}$, $\vec{a}-\vec{b}$

Q.15 In the adjoining diagram vector $\vec{u}-\vec{v}$ is represented by the directed line segment-



(A) $\overrightarrow{BD}$ (B) $\overrightarrow{AC}$

(C) $\overrightarrow{DB}$ (D) $\overrightarrow{CA}$

Q16 If three forces P, Q, R acting on a particle are represented by three sides of a triangle taken in order, then-

(A) $\vec{P}+\vec{Q}+\vec{R}=\vec{0}$ (B) $\vec{P}-\vec{Q}+\vec{R}=\vec{0}$

(C) $\vec{P}+\vec{Q}-\vec{R}=\vec{0}$ (D) $\vec{P}-\vec{Q}-\vec{R}=\vec{0}$

Q.17 If $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$, then unit vector parallel to $\vec{a}+\vec{b}$ is-

(A) $\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$ (B) $\frac{1}{5}(2\hat{i}-\hat{j}+2\hat{k})$

(C) $\frac{1}{\sqrt{3}}(2\hat{i}-\hat{j}+2\hat{k})$ (D) None of these

Q.18 If $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ are two adjacent sides of a parallelogram, then the unit vector along the diagonal determined by these sides is-

(A) $\frac{(3\hat{i}+6\hat{j}-2\hat{k})}{7}$ (B) $\hat{i}+2\hat{j}+8\hat{k}$

(C) $-\hat{i}-2\hat{j}+8\hat{k}$ (D) $\frac{(-\hat{i}-2\hat{j}+8\hat{k})}{\sqrt{69}}$

Question based on: Vectors in terms of position vectors of end points

Q.19 The position vector of a point C with respect to B is $\hat{i}+\hat{j}$ and that of B with respect to A is $\hat{i}-\hat{j}$. The position vector of C with respect to A is-

(A) $2\hat{i}$ (B) $-2\hat{i}$ (C) $2\hat{j}$ (D) $-2\hat{j}$

Q.20 If A, B, C are three points such that $2\overrightarrow{AC}=3\overrightarrow{CB}$, then $2\overrightarrow{OA}+3\overrightarrow{OB}$ equals-

(A) $5\overrightarrow{OC}$ (B) $\overrightarrow{OC}$

(C) $-\overrightarrow{OC}$ (D) None of these

Q.21 If the position vector of the point A and B with respect to point O are respectively $\hat{i}+2\hat{j}-3\hat{k}$ and $-2\hat{i}+3\hat{j}-4\hat{k}$ then $\overrightarrow{BA}$ equals-

(A) $3\hat{i}-\hat{j}+\hat{k}$ (B) $3\hat{i}+\hat{j}-\hat{k}$

(C) $-3\hat{i}+\hat{j}+\hat{k}$ (D) None of these

Question based on: Distance between two points

Q.22 If the end points of $\overrightarrow{AB}$ are (3, –7) and (– 1, – 4), then magnitude of $\overrightarrow{AB}$ is-

(A) 2 (B) 3 (C) 4 (D) 5

Q.23 If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}+3\hat{k}$ then the value of $|\vec{a}+\vec{b}|$ is -

(A) $\sqrt{6}$ (B) $2\sqrt{6}$ (C) $3\sqrt{6}$ (D) $4\sqrt{6}$

Q.24 The vectors $3\hat{i}-2\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}+5\hat{k}$ & $2\hat{i}+\hat{j}-4\hat{k}$ form-

(A) an equilateral triangle

(B) an isosceles triangle

(C) a right angle triangle

(D) None of these

Q.25 If vectors $2\hat{i}+3\hat{j}-2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ represents the adjacent sides of any parallelogram then the length of diagonals of parallelogram are-

(A) $\sqrt{35}, \sqrt{35}$ (B) $\sqrt{35}, \sqrt{11}$

(C) $\sqrt{25}, \sqrt{11}$ (D) None of these

Q.26 If position vectors of the vertices of a triangle are $4\hat{i}+5\hat{j}+6\hat{k}$, $5\hat{i}+6\hat{j}+4\hat{k}$ and $6\hat{i}+4\hat{j}+5\hat{k}$ then this triangle is-

(A) right angled (B) equilateral

(C) isosceles (D) None of these

Q.27 The length of vector $\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{\sqrt{6}}$

(C) 1 (D) None of these

Q.28 If A = (1, 0, 3), B = (3, 1, 5), then 3 kg. wt. along $\overrightarrow{AB}$ is represented by the vector-

(A) $2\hat{i}+2\hat{j}+\hat{k}$ (B) $2\hat{i}+\hat{j}+2\hat{k}$

(C) $\hat{i}+2\hat{j}+2\hat{k}$ (D) $\hat{i}+\hat{j}+\hat{k}$

Q.29 If ℓ_1 and ℓ_2 are lengths of the vectors $\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+5\hat{j}$ respectively, then-

(A) $\ell_1 = \ell_2$ (B) $\ell_1 = -\ell_2$

(C) $\ell_1 < \ell_2$ (D) $\ell_1 > \ell_2$

Q.30 If $\vec{a}=\hat{i}+\lambda\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}+\sqrt{\lambda}\,\hat{k}$ are of equal magnitudes, then value of λ is-

(A) 1 (B) 0

(C) 2 (D) 0 or 1

Question based on **Position vector of dividing point**

Q.31 If the position vector of points A and B with respect to point P are respectively $\vec{a}$ and $\vec{b}$ then the position vector of middle point of $\overrightarrow{AB}$ is -

(A) $\frac{\vec{b}-\vec{a}}{2}$ (B) $\frac{\vec{a}+\vec{b}}{2}$

(C) $\frac{\vec{a}-\vec{b}}{2}$ (D) None of these

Q.32 The position vector of two points P and Q are respectively $\vec{p}$ and $\vec{q}$ then the position vector of the point dividing $\overrightarrow{PQ}$ in 2 : 5 is -

(A) $\frac{\vec{p}+\vec{q}}{2+5}$ (B) $\frac{5\vec{p}+2\vec{q}}{2+5}$

(C) $\frac{2\vec{p}+5\vec{q}}{2+5}$ (D) $\frac{\vec{p}-\vec{q}}{2+5}$

Q.33 The position vector of the vertices of triangle ABC are $\hat{i}$, $\hat{j}$ and $\hat{k}$ then the position vector of its orthocentre is-

(A) $\hat{i}+\hat{j}+\hat{k}$ (B) $2(\hat{i}+\hat{j}+\hat{k})$

(C) $\frac{1}{3}(\hat{i}+\hat{j}+\hat{k})$ (D) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

Q.34 If D, E, F are mid points of sides BC, CA and AB respectively of a triangle ABC, and $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$, $\hat{k}+\hat{i}$ are p. v. of points A, B and C respectively, then p. v. of centroid of ΔDEF is-

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{3}$ (B) $\hat{i}+\hat{j}+\hat{k}$

(C) $2(\hat{i}+\hat{j}+\hat{k})$ (D) $\frac{2(\hat{i}+\hat{j}+\hat{k})}{3}$

Q.35 If D, E and F are midpoints of sides BC, CA and AB of a triangle ABC, then $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$ is equal to-

(A) $\vec{0}$ (B) $2\overrightarrow{BC}$

(C) $2\overrightarrow{AB}$ (D) $2\overrightarrow{CA}$

Q.36 If A, B, C, D be any four points and E and F be the middle points of AC and BD respectively, then $\overrightarrow{AB}+\overrightarrow{CB}+\overrightarrow{CD}+\overrightarrow{AD}$ is equal to-

(A) $3\overrightarrow{EF}$ (B) $3\overrightarrow{FE}$

(C) $4\overrightarrow{EF}$ (D) $4\overrightarrow{FE}$

Q.37 If G is centroid of ΔABC and $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{AC}=\vec{b}$ then $\overrightarrow{AG}$ equals-

(A) $1/2(\vec{a}+\vec{b})$ (B) $1/3(\vec{a}+\vec{b})$

(C) $2/3(\vec{a}+\vec{b})$ (D) $1/6(\vec{a}+\vec{b})$

Q.38 If E is the intersection point of diagonals of parallelogram ABCD and $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}= x\ \overrightarrow{OE}$ then x is equal to (where O represents origin)-

(A) 3 (B) 4

(C) 5 (D) 6

Q.39 If $\vec{a}$, $\vec{b}$, $\vec{c}$ be position vectors of A,B,C respectively and D is the middle point of BC, then $\overrightarrow{AD}$ equals-

(A) $(\vec{b}+\vec{c}-\vec{a})/2$ (B) $(\vec{a}+\vec{c}-2\vec{a})/2$

(C) $(\vec{b}+\vec{c}-2\vec{a})/2$ (D) $(\vec{a}+\vec{b}-2\vec{c})/2$

Q.40 If the position vectors of three consecutive vertices of any parallelogram are respectively $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$, $7\hat{i}+9\hat{j}+11\hat{k}$ then the position vector of its fourth vertex is-

(A) $6(\hat{i}+\hat{j}+\hat{k})$ (B) $7(\hat{i}+\hat{j}+\hat{k})$

(C) $2\hat{j}-4\hat{k}$ (D) $6\hat{i}+8\hat{j}+10\hat{k}$

Q.41 Two points A and P are respectively $\vec{a}+2\vec{b}$ and $\vec{a}$ and P divides AB in the ratio 2: 3 then p.v. of B is-

(A) $\vec{b}$ (B) $\vec{a}-3\vec{b}$

(C) $2\vec{a}-\vec{b}$ (D) $\vec{b}-2\vec{a}$

Q.42 The orthocentre of the triangle whose vertices are $3\hat{i}+2\hat{j}, -2\hat{i}+3\hat{j}$ and $\hat{i}+5\hat{j}$ is-

(A) $\hat{i}+5\hat{j}$ (B) $-2\hat{i}+3\hat{j}$

(C) $3\hat{i}+2\hat{j}$ (D) None of these

Q.43 The centroid of the triangle whose vertices are $\hat{i}+2\hat{j}$, $2\hat{i}+\hat{j}$, $\hat{i}+\hat{j}+\hat{k}$ is-

(A) $4\hat{i}+4\hat{j}+\hat{k}$ (B) $\frac{4\hat{i}+4\hat{j}+\hat{k}}{3}$

(C) $\frac{4\hat{i}+4\hat{j}+\hat{k}}{2}$ (D) None of these

Q.44 If p. v. of vertices of a tetrahedron are $\hat{i}-\hat{j}-\hat{k}, -\hat{i}+\hat{j}-\hat{k}, -\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$, then its centre is-

(A) origin (B) $\hat{i}+\hat{j}+\hat{k}$

(C) $\frac{\hat{i}+\hat{j}+\hat{k}}{4}$ (D) None of these

Q.45 The position vector of the points A and B are $\vec{a}$ and $\vec{b}$ respectively. If P divides AB is the ratio 3 : 1 and Q is the mid point of AP, then the position vector of Q is-

(A) $\frac{\vec{a}+\vec{b}}{2}$ (B) $\frac{\vec{a}-\vec{b}}{2}$

(C) $\frac{5\vec{a}-3\vec{b}}{8}$ (D) $\frac{5\vec{a}+3\vec{b}}{8}$

Question based on **Collinearity of three points**

Q.46 If vectors $(x-2)\hat{i}+\hat{j}$ and $(x+1)\hat{i}+2\hat{j}$ are collinear, then the value of x is-

(A) 3 (B) 4 (C) 5 (D) 0

Q.47 If points $\hat{i}+2\hat{k}$, $\hat{j}+\hat{k}$ and $\lambda\hat{i}+\mu\hat{j}$ are collinear, then-

(A) $\lambda = 2, \mu = 1$ (B) $\lambda = 2, \mu = -1$

(C) $\lambda = -1, \mu = 2$ (D) $\lambda = -1, \mu = -2$

Q.48 If three collinear points A,B,C are such that AB = BC and the position vector of points A and B with respect to origin O are respectively $\vec{a}$ and $\vec{b}$ then the position vector of point C is-

(A) $\frac{\vec{a}-\vec{b}}{2}$ (B) $\frac{\vec{a}+\vec{b}}{2}$

(C) $2\vec{b}-\vec{a}$ (D) None of these

Q.49 If $\vec{a}, \vec{b}$ and $(3\vec{a}-2\vec{b})$ are position vectors of three points, then points are-

(A) collinear

(B) vertices of a right angled triangle

(C) vertices of an equilateral triangle

(D) None of these

Q.50 Three points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear if $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$ when-

(A) $x + y + z = 0$

(B) $x + y + z \neq 0$

(C) $x + y + z$ may or may not be zero

(D) None of these

Q.51 If the vectors $3\hat{i}-2\hat{j}+5\hat{k}$ and $-2\hat{i}+p\hat{j}-q\hat{k}$ are collinear, then (p, q) is-
(A) (4/3, – 10/3) (B) (10, 4/3)
(C) (–4/3, 10/3) (D) (4/3, 10/3)

Q.52 If $A(-\hat{i}+3\hat{j}+2\hat{k})$, $B(-4\hat{i}+2\hat{j}-2\hat{k})$ and $C(5\hat{i}+p\hat{j}+q\hat{k})$ are collinear then the value of p and q respectively-
(A) 5, 10 (B) 10, 5
(C) – 5, 10 (D) 5, –10

Q.53 If the position vectors of the points A, B, C are $3\hat{i}-2\hat{j}+4\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ & $-\hat{i}+4\hat{j}-2\hat{k}$, then A,B,C are-
(A) vertices of a right angled triangle
(B) vertices of an isosceles triangle
(C) vertices of an equilateral triangle
(D) collinear

Q.54 If A, B, C are collinear and their position vector are respectively $\hat{i}-2\hat{j}-8\hat{k}$, $5\hat{i}-2\hat{k}$ & $11\hat{i}+3\hat{j}+7\hat{k}$ then B, divides AC in the ratio-
(A) 1 : 2 (B) 2 : 1
(C) 2 : 3 (D) 3 : 2

Question based on **Relation between two parallel vectors**

Q.55 If $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to sum of the vectors $3\hat{i}+\lambda\hat{j}+2\hat{k}$ and $-2\hat{i}+3\hat{j}+\hat{k}$, then λ equals-
(A) 1 (B) –1 (C) 2 (D) –2

Q.56 If $\vec{a}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=-8\hat{i}+4\hat{j}-6\hat{k}$ are two vectors then $\vec{a}$, $\vec{b}$ are-
(A) like parallel (B) unlike parallel
(C) non-collinear (D) perpendicular

Q.57 If position vectors of A, B, C, D are respectively $2\hat{i}+3\hat{j}+5\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $-5\hat{i}+4\hat{j}-2\hat{k}$ and $\hat{i}+10\hat{j}+10\hat{k}$, then-
(A) $\overrightarrow{AB} \parallel \overrightarrow{CD}$
(B) $\overrightarrow{DC} \parallel \overrightarrow{AD}$
(C) A, B, C are collinear
(D) B, C, D are collinear

Q.58 If $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$ then the unit vector parallel to $\vec{a}+\vec{b}$, is-
(A) $\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$ (B) $\frac{1}{5}(2\hat{i}-\hat{j}+2\hat{k})$
(C) $\frac{1}{\sqrt{3}}(2\hat{i}-\hat{j}+2\hat{k})$ (D) None of these

Q.59 If $\vec{A}=(x+1)\vec{a}+(2y-3)\vec{b}$ and $\vec{B}=5\vec{a}-2\vec{b}$ are two vectors such that $2\vec{A}=3\vec{B}$ & $\vec{a}$, $\vec{b}$ are non zero non-collinear vectors then-
(A) x = 13/2, y = 0 (B) x = 0, y = 3
(C) x = –13/2, y = 0 (D) None of these

Q.60 The p. v. of four points A, B, C, D are respectively $2\hat{i}+\hat{j}$, $\hat{i}-3\hat{j}$, $3\hat{i}+2\hat{j}$ and $\hat{i}+\lambda\hat{j}$. If $\overrightarrow{AB} \parallel \overrightarrow{CD}$, then value of λ is-
(A) 6 (B) – 6 (C) 8 (D) – 8

Question based on **Coplanar and non-coplanar vectors**

Q.61 If $\vec{p}=2\vec{a}-3\vec{b}$, $\vec{q}=\vec{a}-2\vec{b}+\vec{c}$, $\vec{r}=-3\vec{a}+\vec{b}+2\vec{c}$, $\vec{a}$, $\vec{b}$, $\vec{c}$ being non zero, non coplanar vectors then the vectors $-2\vec{a}+3\vec{b}-\vec{c}$ is equal to -
(A) $\frac{-7\vec{q}+\vec{r}}{5}$ (B) $\vec{p}-4\vec{q}$
(C) $2\vec{p}-3\vec{q}+\vec{r}$ (D) $4\vec{p}-2\vec{r}$

Q.62 If the position vectors of four points P, Q, R, S respectively $2\vec{a}+4\vec{c}$, $5\vec{a}+3\sqrt{3}\vec{b}+4\vec{c}$, $-2\sqrt{3}\vec{b}+\vec{c}$ and $2\vec{a}+\vec{c}$ then-
(A) $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ (B) $\overrightarrow{PQ}=\overrightarrow{RS}$
(C) $\overrightarrow{PQ} \neq \overrightarrow{RS}$ (D) None of these

Q.63 If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are four linearly independent vectors and $x\vec{a}+y\vec{b}+z\vec{c}+u\vec{d}=\vec{0}$, then-
(A) x + y + z + u = 0 (B) x + y = z + u
(C) x + z = y + u (D) All correct

Q.64 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar vectors then the three points whose position vector are $\vec{a} - 2\vec{b} + 3\vec{c}$, $2\vec{a} + m\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are collinear, if m equals-

(A) 2 (B) 3 (C) 0 (D) 1

Question based on — Scalar or Dot product of two vectors

Q.65 If the angle between $\vec{a}$ and $\vec{b}$ is θ then for $\vec{a}.\vec{b} \geq 0$

(A) $0 \leq \theta \leq \pi$ (B) $0 < \theta < \pi/2$

(C) $\pi/2 \leq \theta \leq \pi$ (D) $0 \leq \theta \leq \pi/2$

Q.66 If the moduli of vectors $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a}.\vec{b} = 1$, then the angle θ between them is-

(A) $\theta = \pi/6$ (B) $\theta = \pi/3$

(C) $\theta = \pi/2$ (D) $\theta = 2\pi/3$

Q.67 If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ & $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$ then the projection of $\vec{a} + \vec{b}$ on $\vec{c}$ is-

(A) 17/3 (B) 5/3

(C) 4/3 (D) None of these

Q.68 If $\vec{a}$ and $\vec{b}$ are unit vectors and 60° is the angle between them, then $(2\vec{a} - 3\vec{b}) \cdot (4\vec{a} + \vec{b})$ equals-

(A) 5 (B) 0

(C) 11 (D) None of these

Q.69 If vectors $3\hat{i} + 2\hat{j} + 8\hat{k}$ and $2\hat{i} + x\hat{j} + \hat{k}$ are perpendicular then x is equal to-

(A) 7 (B) –7 (C) 5 (D) –4

Q.70 If vector $\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$ and $2\vec{b} + \vec{a}$ is perpendicular to $\vec{a}$, then-

(A) $|\vec{a}| = \sqrt{2}\,|\vec{b}|$ (B) $|\vec{a}| = 2|\vec{b}|$

(C) $|\vec{b}| = \sqrt{2}\,|\vec{a}|$ (D) $|\vec{a}| = |\vec{b}|$

Q.71 If $|\vec{a}| = |\vec{b}|$, then $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ is-

(A) positive (B) negative

(C) zero (D) None of these

Q.72 If $\vec{a}$ and $\vec{b}$ are vectors of equal magnitude 2 and α be the angle between them, then magnitude of $(\vec{a} + \vec{b})$ will be 2 if -

(A) $\alpha = \pi/3$ (B) $\alpha = \pi/4$

(C) $\alpha = \pi/2$ (D) $\alpha = 2\pi/3$

Q.73 If $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}$, then $(2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b})$ equals-

(A) 14 (B) –14

(C) 0 (D) None of these

Q.74 Angle between the vectors $2\hat{i} + 6\hat{j} + 3\hat{k}$ and $12\hat{i} - 4\hat{j} + 3\hat{k}$ is -

(A) $\cos^{-1}\left(\frac{1}{10}\right)$ (B) $\cos^{-1}\left(\frac{9}{11}\right)$

(C) $\cos^{-1}\left(\frac{9}{91}\right)$ (D) $\cos^{-1}\left(\frac{1}{9}\right)$

Q.75 If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ be p.v. of four points A,B,C and D respectively, then the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is-

(A) π/4 (B) π/2

(C) π (D) None of these

Q.76 If the force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ moves a particle from $\hat{i} + \hat{j} - \hat{k}$ to $2\hat{i} - \hat{j} + \hat{k}$, then the work done is-

(A) 6 (B) 5 (C) 4 (D) 3

Q.77 Two forces $P = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $Q = -\hat{i} + 2\hat{j} - \hat{k}$ are acting on a particle. These forces displace the particle from point A$(4\hat{i} - 3\hat{j} - 2\hat{k})$ to point B $(6\hat{i} + \hat{j} - 3\hat{k})$. The work done by these forces is-

(A) 15 units (B) –15 units

(C) 10 units (D) –10 units

Q.78 The projection of vector $\hat{i} + 2\hat{j} + 2\hat{k}$ on x- axis is -

(A) 2 (B) 1 (C) $\sqrt{5}$ (D) 3

Q.79 $\vec{a}$ and $\vec{b}$ are vectors of equal magnitude and angle between them is 120º. If $\vec{a}.\vec{b} = -8$, then $|\vec{a}|$ equals-

(A) 4 (B) –4 (C) 5 (D) –5

Q.80 If the points P, Q, R, S are respectively $\hat{i}-\hat{k}$, $-\hat{i}+2\hat{j}, 2\hat{i}-3\hat{k}$ and $3\hat{i}-2\hat{j}-\hat{k}$, then projection of $\overrightarrow{PQ}$ on $\overrightarrow{RS}$ is-

(A) 4/3 (B) – 4/3 (C) 3/4 (D) –3/4

Q.81 If angle between vectors $\vec{a}$ and $\vec{b}$ is 120º and $|\vec{a}| = 3, |\vec{b}| = 4$, then length of $4\vec{a} - 3\vec{b}$ is-

(A) $12\sqrt{3}$ (B) $2\sqrt{3}$

(C) 432 (D) None of these

Q.82 Vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular, when-

(A) $\vec{a} = \vec{0}$ (B) $\vec{a}+\vec{b} = \vec{0}$ or $\vec{a}-\vec{b} = \vec{0}$

(C) $\vec{b} = \vec{0}$ (D) None of these

Q.83 If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then -

(A) $\vec{a}$ and $\vec{b}$ are perpendicular

(B) $\vec{a}, \vec{b}$ are parallel to each other

(C) $\vec{a} \neq \vec{0}$

(D) $\vec{b} \neq \vec{0}$

Q.84 If the angle between two vectors $\vec{a}$ and $\vec{b}$ is 120º. If $|\vec{a}| = 2$, $|\vec{b}| = 1$ then the value of $|2\vec{a}+\vec{b}|$ is-

(A) $\sqrt{21}$ (B) $\sqrt{13}$

(C) 21 (D) 13

Q.85 For any vector $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, $(\vec{r}.\hat{i})\hat{i}+(\vec{r}.\hat{j})\hat{j}+(\vec{r}.\hat{k})\hat{k}$ equals-

(A) $\vec{0}$ (B) $2\vec{r}$

(C) $\vec{r}$ (D) $3\vec{r}$

Q.86 If $\vec{a}$ and $\vec{b}$ be two non- zero vectors, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ equals-

(A) $|\vec{a}+\vec{b}|$ (B) $|\vec{a}-\vec{b}|^2$

(C) $|\vec{a}+\vec{b}|^2$ (D) $|\vec{a}|^2 - |\vec{b}|^2$

Q.87 If sum of two unit vectors is again a unit vector, then modulus of their difference is-

(A) 1 (B) 2

(C) $\sqrt{2}$ (D) $\sqrt{3}$

Q.88 The angle between $(\hat{i}+\hat{j})$ and $(\hat{i}+\hat{k})$ is-

(A) 0 (B) π/4 (C) π/2 (D) π/3

Q.89 The angle between the vectors $3\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}-2\hat{j}+4\hat{k}$ is-

(A) $\sin^{-1}\frac{2}{\sqrt{5}}$ (B) $\sin^{-1}\frac{2}{\sqrt{7}}$

(C) $\cos^{-1}\frac{2}{\sqrt{5}}$ (D) $\cos^{-1}\frac{2}{\sqrt{7}}$

Q.90 If $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$, then the angle between vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is-

(A) 0º (B) 30º (C) 60º (D) 90º

Q.91 If angle between vectors $\vec{a}$ and $\vec{b}$ is 30º, then angle between $3\vec{a}$ and $4\vec{b}$ will be-

(A) 60º (B) 30º (C) 0º (D) 90º

Q.92 The unit vector which bisect the angle between $\hat{i}$ and $\hat{j}$ is-

(A) $\hat{k}$ (B) $\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(C) $\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$ (D) None of these

Q.93 If $\vec{a} = 4\hat{i}+6\hat{j}$ and $\vec{b} = 3\hat{j}+4\hat{k}$, then component vector of $\vec{a}$ along $\vec{b}$ is-

(A) $\frac{18(3\hat{j}+4\hat{k})}{10\sqrt{3}}$ (B) $\frac{18(3\hat{j}+4\hat{k})}{25}$

(C) $\frac{18(3\hat{j}+4\hat{k})}{\sqrt{13}}$ (D) $3\hat{i}+4\hat{k}$

Q.94 A force $\vec{F} = \hat{i}-3\hat{j}+5\hat{k}$ acting on a particle displaces it from point A(4, – 3, –2) to B (6,1, – 3) then the work done by the force is-

(A) –15 unit (B) 16 unit

(C) 0 (D) None of these

Q.95 If by acting three forces $\vec{F}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{F}_2 = -\hat{i} + 2\hat{j} - \hat{k}$, $\vec{F}_3 = -\hat{j} - \hat{k}$ on a particle it displaces it from point A(4, –3, –2) to point B (6, 1, – 3) then the work done by the force is-

(A) 1 unit (B) 2 unit
(C) 0 unit (D) None of these

Q.96 The work done in moving an object along the vector $3\hat{i} + 2\hat{j} - 5\hat{k}$, if the applied force is $F = 2\hat{i} - \hat{j} - \hat{k}$ is-

(A) 7 (B) 8
(C) 9 (D) 10

Q.97 If angle between two unit vectors $\vec{a}$ and $\vec{b}$ is θ then sin (θ/2) is equal to-

(A) $2\,|\vec{a} - \vec{b}|$ (B) $\frac{1}{2}|\vec{a} - \vec{b}|$
(C) $\frac{1}{2}\,|\vec{a} + \vec{b}|$ (D) $2\,(\vec{a} + \vec{b})$

Question based on Vector or cross product of two vectors

Q.98 If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 3\hat{k}$ then $|\vec{a} \times \vec{b}|$ is

(A) $\sqrt{6}$ (B) $2\sqrt{6}$ (C) $\sqrt{70}$ (D) $4\sqrt{6}$

Q.99 If $\vec{a}$ and $\vec{b}$ are two vectors, then-

(A) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$ (B) $|\vec{a} \times \vec{b}| \leq |\vec{a}||\vec{b}|$
(C) $|\vec{a} \times \vec{b}| > |\vec{a}||\vec{b}|$ (D) $|\vec{a} \times \vec{b}| < |\vec{a}||\vec{b}|$

Q.100 If θ be the angle between vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} + 2\hat{j} + \hat{k}$, then the value of sin θ is-

(A) $\sqrt{6/7}$ (B) $\frac{2\sqrt{6}}{7}$
(C) 1/7 (D) None of these

Q.101 If $|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is -

(A) 0º (B) 90º
(C) 60º (D) 45º

Q.102 The unit vector perpendicular to vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is-

(A) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (B) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$
(C) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (D) None of these

Q.103 If $|\vec{a}.\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 4$, then the angle between $\vec{a}$ and $\vec{b}$ is-

(A) $\cos^{-1} 3/4$ (B) $\cos^{-1} 3/5$
(C) $\sin^{-1} 4/5$ (D) π/4

Q.104 If $|(\vec{a} \times \vec{b})|^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to -

(A) 3 (B) 8
(C) 12 (D) 16

Q.105 $(\hat{i} + \hat{j}).[(\hat{j} + \hat{k}) \times (\hat{k} + \hat{i})]$ equals-

(A) 0 (B) 1
(C) –1 (D) 2

Q.106 If for vectors $\vec{a}$ & $\vec{b}$, $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a}.\vec{b} = 0$, then-

(A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$
(C) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ (D) None of these

Q.107 In a parallelogram PQRS, $\overrightarrow{PQ} = \vec{a} + \vec{b}$ and $\overrightarrow{PR} = \vec{a} - \vec{b}$, then its vector area is-

(A) $|\vec{a}|^2 - |\vec{b}|^2$ (B) $\vec{a} \times \vec{b}$
(C) $2(\vec{a} \times \vec{b})$ (D) $2(\vec{b} \times \vec{a})$

Q.108 If the diagonals of a parallelogram are respectively $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$, then the area of parallelogram is-

(A) $\sqrt{14}$ (B) $2\sqrt{14}$
(C) $2\sqrt{6}$ (D) $\sqrt{38}$

Q.109 If adjacent sides of a triangle are represented by vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$, then vector area is -

(A) 13/2 (B) 41/2
(C) 41 (D) None of these

Q.110 If $\hat{i}-\hat{j}+2\hat{k}$, $2\hat{i}+\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$ are position vectors of vertices of a triangle, then its area is-

(A) 26 (B) 13

(C) $2\sqrt{13}$ (D) $\sqrt{13}$

Q.111 Two constant forces P = $2\hat{i}-5\hat{j}+6\hat{k}$ and $Q=-\hat{i}+2\hat{j}-\hat{k}$ are acting on a point A (4,–3,–2). The moment of their resultant about origin (0, 0, 0) is-

(A) $21\hat{i}+22\hat{j}+9\hat{k}$ (B) $-(21\hat{i}+22\hat{j}+9\hat{k})$

(C) $21\hat{i}-22\hat{j}-9\hat{k}$ (D) None of these

Q.112 If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$ & $\vec{c}=\hat{i}+\hat{j}+\hat{k}$, then $(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$ equals-

(A) 60 (B) 64

(C) 74 (D) –74

Q.113 Vector $\vec{a}\times(\vec{b}+\vec{a})$ is perpendicular to-

(A) both $\vec{a}$ and $\vec{b}$ (B) $\vec{a}$

(C) $\vec{b}$ (D) Neither $\vec{a}$ nor $\vec{b}$

Q.114 If angle between vector $\vec{a}$ and $\vec{b}$ lies between π/2 and 3π/4, then -

(A) $|\vec{a}\times\vec{b}| \le |\vec{a}.\vec{b}|$ (B) $|\vec{a}\times\vec{b}| \ge |\vec{a}.\vec{b}|$

(C) $|\vec{a}\times\vec{b}| < |\vec{a}.\vec{b}|$ (D) $|\vec{a}\times\vec{b}| > |\vec{a}.\vec{b}|$

Q.115 If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$, then unit vector along the direction of the resultant is-

(A) $3\hat{i}+5\hat{j}+4\hat{k}$ (B) $\frac{3\hat{i}+5\hat{j}+4\hat{k}}{50}$

(C) $\frac{3\hat{i}+5\hat{j}+4\hat{k}}{5\sqrt{2}}$ (D) None of these

Q.116 If points P(1, –1, 2), Q(2, 0, –1) and R (0, 2, 1) be any three points, then unit vector perpendicular to the plane PQR is-

(A) $2\hat{i}+\hat{j}+\hat{k}$ (B) $\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$

(C) $\frac{3\hat{i}+2\hat{j}-\hat{k}}{\sqrt{14}}$ (D) None of these

Q.117 Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ & $\vec{c}=-\hat{i}+2\hat{j}+\hat{k}$, then the unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is-

(A) $\hat{i}$ (B) $\hat{j}$

(C) $\frac{\hat{k}+\hat{i}}{\sqrt{2}}$ (D) $(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$

Q.118 If $|\vec{a}|$ = 10, $|\vec{b}|$ = 2 and $\vec{a}.\vec{b}$ = 12, then $|\vec{a}\times\vec{b}|$ equals-

(A) 16 (B) 8

(C) 32 (D) None of these

Q.119 Which one of the following is correct-

(A) $\hat{i}.\hat{i}+\hat{j}.\hat{j}+\hat{k}.\hat{k}=0$

(B) $\hat{i}\times\hat{j}+\hat{j}\times\hat{k}+\hat{k}\times\hat{i}=\vec{0}$

(C) $\hat{i}.\hat{i}+\hat{j}.\hat{j}+\hat{k}.\hat{k}=3$

(D) $\hat{i}\times\hat{j}+\hat{j}\times\hat{k}+\hat{k}\times\hat{i}=3$

Q.120 If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, $\vec{a}\neq\vec{0}$, then-

(A) $\vec{b}=\vec{0}$ (B) $\vec{b}=\vec{c}$

(C) $\vec{b}\neq\vec{c}$ (D) None of these

Q.121 For any three vectors $\vec{a}, \vec{b}, \vec{c}$, $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$ equals-

(A) $\vec{a}+\vec{b}+\vec{c}$ (B) $[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $\vec{a}\times\vec{b}\times\vec{c}$ (D) $\vec{0}$

Q.122 $|(2\hat{i}+\hat{k})\times(\hat{i}+\hat{j}+\hat{k})|$ is equal to-

(A) 6 (B) $\sqrt{6}$

(C) 3 (D) $\sqrt{3}$

Q.123 $(2\vec{a}+3\vec{b})\times(5\vec{a}+7\vec{b})$ is equal to-

(A) $\vec{a}+\vec{b}$ (B) $\vec{b}\times\vec{a}$

(C) $\vec{a}\times\vec{b}$ (D) $7\vec{a}+10\vec{b}$

Q.124 For any two vectors $\vec{a}$, $\vec{b}$ $\{|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2\}+|\vec{a}|^2|\vec{b}|^2$ equals-

(A) $|\vec{a}|^2|\vec{b}|^2$ (B) $2|\vec{a}|^2|\vec{b}|^2$

(C) 0 (D) None of these

Q.125 If vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $-3\hat{i}+2\hat{j}+\hat{k}$ represent adjacent sides of a parallelogram, then its area is-

(A) $5\sqrt{6}$ (B) $6\sqrt{2}$

(C) $6\sqrt{5}$ (D) 180

Q.126 If $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=-2\hat{j}-\hat{k}$ then the area of the parallelogram with diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ will be-

(A) $\sqrt{21}$ (B) $2\sqrt{21}$

(C) $\frac{1}{2}\sqrt{21}$ (D) None of these

Q.127 If A (1, –1, 2), B(2, 1, –1), C(3, –1, 2) be any three points, then area of ABC is-

(A) $\sqrt{13}$ (B) $2\sqrt{13}$

(C) $\frac{1}{2}\sqrt{3}$ (D) None of these

Q.128 If the vertices of any triangle are $\hat{i}, \hat{j}, \hat{k}$ then its area is -

(A) 1 unit (B) 2 unit

(C) $\sqrt{2}$ unit (D) $\frac{\sqrt{3}}{2}$ unit

Q.129 If $\hat{i}+2\hat{j}+3\hat{k}$, $-\hat{i}-\hat{j}+8\hat{k}$, $-4\hat{i}+4\hat{j}+6\hat{k}$ be p.v. of A, B, and C respectively, then ΔABC is-

(A) right angled (B) isosceles

(C) equilateral (D) None of these

Q.130 A force $F=2\hat{i}+\hat{j}-\hat{k}$ acts at a point A whose position vector is $2\hat{i}-\hat{j}$. The moment of F about origin is-

(A) $\hat{i}+2\hat{j}+4\hat{k}$ (B) $\hat{i}-2\hat{j}+4\hat{k}$

(C) $\hat{i}+2\hat{j}-4\hat{k}$ (D) $\hat{i}-2\hat{j}-4\hat{k}$

Q.131 A force $F=3\hat{i}+\hat{k}$ passing through A whose position vector is $2\hat{i}-\hat{j}+3\hat{k}$, then the moment of the force about point P whose position vector is, $\hat{i}+2\hat{j}-\hat{k}$ is-

(A) $-3\hat{i}+11\hat{j}+9\hat{k}$ (B) $2\hat{i}+10\hat{j}+8\hat{k}$

(C) $\hat{i}+3\hat{j}+7\hat{k}$ (D) $4\hat{i}+3\hat{j}-6\hat{k}$

Question based on Scalar Triple product

Q.132 If $[3\hat{i}\;\; 5\hat{j}-3\hat{k}\;\; \lambda\hat{i}+\hat{k}] = 5$, then the value of λ is-

(A) 1 (B) 2

(C) 3 (D) Not possible

Q.133 If $\vec{a}=4\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=3\hat{i}+2\hat{j}-\hat{k}$ & $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$ represent three coterminous edges of a parallelopiped then its volume is-

(A) 60 (B) 15 (C) 30 (D) 40

Q.134 $[(\hat{i}\times\hat{j})\times(\hat{i}\times\hat{k})].\hat{j}$ equals-

(A) 1 (B) –1

(C) 0 (D) None of these

Q.135 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular unit vectors, then $[\vec{a}\;\vec{b}\;\vec{c}]$ equals-

(A) 0 (B) ±1

(C) 3 (D) 1

Q.136 $[\vec{a}\;\vec{b}\;\vec{c}]$ will not be zero when-

(A) $\vec{a}=\vec{b}=\vec{c}$

(B) $\vec{a}=\vec{b}$ or $\vec{b}=\vec{c}$

(C) $\vec{a}, \vec{b}, \vec{c}$ are coplanar

(D) $\vec{a}\perp\vec{b}$ or $\vec{b}\perp\vec{c}$

Q.137 The vector $\vec{a}$ which is collinear with the vector $\vec{b}=2\hat{i}-\hat{j}$ and $\vec{a}.\vec{b}=10$ is-

(A) $4\hat{i}-2\hat{j}$ (B) $-2\hat{i}+4\hat{j}$

(C) $2\hat{i}+4\hat{j}+\hat{k}$ (D) $4\hat{i}+2\hat{j}-\hat{k}$

Q.138 Three vectors $\hat{i}-\hat{j}-\hat{k}$, $-\hat{i}+\hat{j}-\hat{k}$ & $-\hat{i}-\hat{j}+\hat{k}$ are-

(A) coplanar

(B) non- coplanar

(C) two are perpendicular to each other

(D) none of these

Q.139 If the volume of the tetrahedron with edges $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+a\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$ is 6 cubic units, then a is-

(A) 1 (B) –1 (C) 2 (D) –17

Q.140 If $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+2\hat{k}$ then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to -

(A) 10 (B) 7 (C) 24 (D) 6

Q.141 If $\vec{a},\vec{b},\vec{c}$ are any three coplanar unit vectors then -

(A) $\vec{a}.(\vec{b}\times\vec{c})=1$ (B) $\vec{a}.(\vec{b}\times\vec{c})=3$

(C) $(\vec{a}\times\vec{b}).\vec{c}=0$ (D) $(\vec{c}\times\vec{a}).\vec{b}=1$

Q.142 If vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+p\hat{k}$ are coplanar, then the value of p is

(A) 1 (B) 2 (C) – 1 (D) – 2

Q.143 If $\vec{a},\vec{b},\vec{c}$ are three non- zero coplanar vectors so that $\vec{a}.\vec{b}=0$ and $\vec{b}.\vec{c}=0$, then-

(A) $\vec{a}.\vec{c}=0$ (B) $\vec{a}.\vec{c}\neq 0$

(C) $\vec{a}.\vec{c}>0$ (D) None of these

Q.144 For any non-zero vector $\vec{d}$; $\vec{d}.\vec{a}=\vec{d}.\vec{b}=\vec{d}.\vec{c}=0$ then $[\vec{a}\ \vec{b}\ \vec{c}]$ equals -

(A) 0 (B) 1

(B) – 1 (D) None of these

Q.145 If $[2\hat{i}\quad \hat{j}+\hat{k}\quad \lambda\hat{i}-2\hat{k}]=-4$ then λ is equal to-

(A) –1 (B) 1

(C) 2 (D) any real number

Q.146 If $\vec{a},\vec{b},\vec{c}$ are coplanar vectors, then which of the following are non-coplanar vectors-

(A) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$

(B) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$

(C) $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}-\vec{a}$

(D) None of these

Q.147 If four points A(1, 2, –1), B(0, 1, m), C (–1, 2, 1), D(2, 1, 3) are coplanar, then the value of m is-

(A) 2 (B) 0 (C) 5 (D) – 5

Q.148 A unit vector which is coplanar with vector $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is-

(A) $\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (B) $\frac{(\hat{j}-\hat{k})}{\sqrt{2}}$

(C) $\frac{(\hat{k}-\hat{j})}{\sqrt{2}}$ (D) $\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

Q.149 Four points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are coplanar if-

(A) $[\vec{a}\ \vec{b}\ \vec{c}]=0$

(B) $[\vec{b}\ \vec{c}\ \vec{d}]=0$

(C) $[\vec{a}-\vec{d}\ \ \vec{b}-\vec{d}\ \ \vec{c}-\vec{d}]=0$

(D) None of these

Q.150 If p.v. of vertices A, B, C with respect to vertex O of any tetrahedron are $6\hat{i}, 6\hat{j}, \hat{k}$ respectively, then its volume is-

(A) 1/3 (B) 1/6

(C) 3 (D) 6

Q.151 If volume of a tetrahedron is 5 units and vertices are A (2, 1, –1), B(3, 0, 1), C(2, –1, 3) and fourth vertex is on y- axis, then its coordinates are-

(A) (0, 8, 0)

(B) (0, – 7, 0)

(C) (0, 8, 0), (0, – 7, 0)

(D) None of these

Q.152 If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are position vectors of four vertices of a tetrahedron, then its volume is-

(A) (1/2) $[\vec{a}-\vec{d}\ \ \vec{b}-\vec{d}\ \ \vec{c}-\vec{d}]$

(B) (1/3) $[\vec{a}-\vec{d}\ \ \vec{b}-\vec{d}\ \ \vec{c}-\vec{d}]$

(C) (1/4) $[\vec{a}-\vec{d}\ \ \vec{b}-\vec{d}\ \ \vec{c}-\vec{d}]$

(D) (1/6) $[\vec{a}-\vec{d}\ \ \vec{b}-\vec{d}\ \ \vec{c}-\vec{d}]$

Question based on **Vector triple product**

Q.153 If $\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ & $\vec{c}=\hat{i}+3\hat{j}-\hat{k}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to-

(A) $20\hat{i}-3\hat{j}+7\hat{k}$ (B) $20\hat{i}+3\hat{j}+7\hat{k}$

(C) $20\hat{i}+3\hat{j}-7\hat{k}$ (D) None of these

Q.154 $\vec{a}\times(\vec{b}\times\vec{c})$ is coplanar with-

(A) $\vec{a}$ and $\vec{b}$ (B) $\vec{b}$ and $\vec{c}$

(C) $\vec{c}$ and $\vec{a}$ (D) None of these

Q.155 For three vectors $\vec{a}, \vec{b}, \vec{c}$ correct statement is-

(A) $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}.(\vec{a}\times\vec{c})$

(B) $(\vec{a}\times\vec{b}).\vec{c} = \vec{a}.(\vec{b}\times\vec{c})$

(C) $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$

(D) None of these

Q.156 The value of

$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$ is-

(A) $\vec{0}$ (B) 1

(C) $\vec{a}+\vec{b}+\vec{c}$ (D) 2 $[\vec{a}\ \vec{b}\ \vec{c}]$

Q.157 If $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$, then it is possible that-

(A) $\vec{a}\perp\vec{b}$ (B) $\vec{a}\perp\vec{c}$

(C) $\vec{a}\parallel\vec{c}$ (D) $\vec{b}\parallel\vec{c}$

Q.158 For any vectors $\vec{a}, \vec{b}, \vec{c}$ correct statement is-

(A) $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$

(B) $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$

(C) $\vec{a}.(\vec{b}\times\vec{c}) = \vec{a}.\vec{b}\times\vec{a}.\vec{c}$

(D) $\vec{a}.(\vec{b}-\vec{c}) = \vec{a}.\vec{b}-\vec{a}.\vec{c}$

Q.159 $[\vec{a}\quad \vec{b}\quad \vec{a}\times\vec{b}]$ equals-

(A) $|\vec{a}\times\vec{b}|$ (B) $|\vec{a}\times\vec{b}|^2$

(C) $|\vec{a}\,.\,\vec{b}|$ (D) $|\vec{a}|\,|\vec{b}|$

Q.160 Which of the following is true statement-

(A) $(\vec{a}\times\vec{b})\times\vec{c}$ is coplanar with $\vec{c}$

(B) $(\vec{a}\times\vec{b})\times\vec{c}$ is perpendicular to $\vec{a}$

(C) $(\vec{a}\times\vec{b})\times\vec{c}$ is perpendicular to $\vec{b}$

(D) $(\vec{a}\times\vec{b})\times\vec{c}$ is perpendicular to $\vec{c}$

Q.161 $\hat{j}\times(\hat{j}\times\hat{k})$ equals-

(A) $\hat{i}$ (B) $-\hat{i}$

(C) $\hat{k}$ (D) $-\hat{k}$

Q.162 $(\vec{a}\times\vec{b})\times\vec{c}$ equals-

(A) $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$ (B) $(\vec{a}.\vec{b})\vec{c}-(\vec{a}.\vec{c})\vec{b}$

(C) $(\vec{b}.\vec{c})\vec{a}-(\vec{a}.\vec{c})\vec{b}$ (D) $(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}$

Q.163 $(\hat{i}\times\hat{j}).[(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i})]$ equals-

(A) 0 (B) 1

(C) –1 (D) 2

LEVEL- 2

Q.1 If C is mid point of AB and P is any point outside AB, then-

(A) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(B) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

(C) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

(D) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$

Q.2 If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j}$, $\vec{c} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, $\vec{d} = \hat{k} - \hat{j}$, then the ratio of the magnitudes of vectors $(\vec{b} - \vec{a})$ and $(\vec{d} - \vec{c})$ is-

(A) 1 : 2 (B) 2 : 1
(C) 1 : 3 (D) 1 : 4

Q.3 If vector $\overrightarrow{AB} = 3\hat{i} - 3\hat{k}$, $\overrightarrow{AC} = \hat{i} - 2\hat{j} + \hat{k}$ represents the sides of any triangle ABC then the length of median AM is-

(A) $\sqrt{6}$ (B) $\sqrt{3}$
(C) $2\sqrt{3}$ (D) $3\sqrt{2}$

Q.4 If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are position vectors of the points A, B, C and D such that $\vec{a} + \vec{c} = \vec{b} + \vec{d}$, then ABCD is a-

(A) parallelogram (B) square
(C) rectangle (D) Trapezium

Q.5 If A, B, P, Q, R be any five points in a plane and forces $\overrightarrow{AP}$, $\overrightarrow{AQ}$, $\overrightarrow{AR}$ act at the point A and forces $\overrightarrow{PB}$, $\overrightarrow{QB}$, $\overrightarrow{RB}$ act at the point B, then their resultant is-

(A) $3\overrightarrow{AB}$ (B) $3\overrightarrow{BA}$
(C) $3\overrightarrow{PQ}$ (D) $3\overrightarrow{PR}$

Q.6 If $|\vec{b}|$ = 10, then the vector b which is collinear with the vector $2\sqrt{2}\,\hat{i} - \hat{j} + 4\hat{k}$ is-

(A) $4\sqrt{2}\,\hat{i} - 2\hat{j} + 8\hat{k}$ (B) $-4\sqrt{2}\,\hat{i} - 2\hat{j} + 8\hat{k}$
(C) $4\sqrt{2}\,\hat{i} + 2\hat{j} + 8\hat{k}$ (D) None of these

Q.7 The mid point of points which divide line joining the points $\vec{a}$ and $\vec{b}$ in the ratio 1: 2 and 2 : 1 is-

(A) $\vec{a} + \vec{b}$ (B) $\dfrac{\vec{a} + \vec{b}}{2}$
(C) $\dfrac{\vec{a} + \vec{b}}{3}$ (D) None of these

Q.8 If $\vec{a} + 5\vec{b} = \vec{c}$ and $\vec{a} - 7\vec{b} = 2\vec{c}$, then-

(A) $\vec{a}$ and $\vec{c}$ are like but $\vec{b}$ and $\vec{c}$ are unlike vectors

(B) $\vec{a}$ and $\vec{b}$ are unlike vectors and so also $\vec{a}$ and $\vec{c}$

(C) $\vec{b}$ and $\vec{c}$ are like but $\vec{a}$ and $\vec{b}$ are unlike vectors

(D) $\vec{a}$ and $\vec{c}$ are unlike vectors and so also $\vec{b}$ and $\vec{c}$

Q.9 If p. v. of vertices of a ΔABC are $2\hat{i} + 4\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$, $3\hat{i} + 6\hat{j} - 3\hat{k}$, then which of the following angles is a right angle-

(A) $\angle A$ (B) $\angle B$
(C) $\angle C$ (D) None of these

Q.10 $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non zero vectors no two of them are parallel. If $\vec{a} + \vec{b}$ is collinear to $\vec{c}$ and $\vec{b} + \vec{c}$ is collinear to $\vec{a}$, then $\vec{a} + \vec{b} + \vec{c}$ is equal to-

(A) $\vec{a}$ (B) $\vec{b}$
(C) $\vec{c}$ (D) None of these

Q.11 If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 2\hat{k}$ & $\vec{c} = 2\hat{i} - \hat{j} + 5\hat{k}$ are vectors, then the vectors $\vec{a}, \vec{b}, \vec{c}$ are-

(A) linearly independent
(B) collinear
(C) linearly dependent
(D) None of these

Q.12 If two forces acting at a point are represented by $n\overrightarrow{OP}$ and $m\overrightarrow{OQ}$ and their resultant is represented by $(m + n)\ \overrightarrow{OR}$, then R is a point such that-

(A) m : n = RQ : PR

(B) m : n = PR : RQ

(C) R is the midpoint of PQ

(D) None of these

Q.13 If $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$ are the position vectors of the vertices A, B and C respectively of triangle ABC. The position vector of the point where the bisector of angle A meets BC is-

(A) $\frac{2}{3}(-3\hat{i}-4\hat{j}-3\hat{k})$

(B) $\frac{6\hat{i}+13\hat{j}+18\hat{k}}{3}$

(C) $\frac{2}{3}(6\hat{i}+8\hat{j}+6\hat{k})$

(D) $-\frac{2}{3}(6\hat{i}+8\hat{j}+6\hat{k})$

Q.14 If $\vec{p}$, $\vec{q}$, $\vec{r}$, $\vec{s}$ are position vectors of points P, Q, R, S such that $\vec{p}-\vec{q} = 2(\vec{s}-\vec{r})$, then-

(A) PQ and RS bisect each other

(B) QS and PR bisect each other

(C) PQ and RS divide each other in 2 : 1

(D) QS and PR divide each other in 2 : 1

Q.15 ABCDE is a pentagon. Force $\overrightarrow{AB}$, $\overrightarrow{AE}$, $\overrightarrow{DC}$, $\overrightarrow{ED}$ act at a point. Which force should be added to this system to make the resultant $2\ \overrightarrow{AC}$ -

(A) $\overrightarrow{AC}$ (B) $\overrightarrow{BC}$

(C) $\overrightarrow{BD}$ (D) $\overrightarrow{AD}$

Q.16 If G and G′ be centroides of triangles ABC and A′B′C′. Then $\overrightarrow{AA'}+\overrightarrow{BB'}+\overrightarrow{CC'}$ is equal to -

(A) $\overrightarrow{GG'}$ (B) $2\overrightarrow{GG'}$

(C) $3\overrightarrow{GG'}$ (D) $\frac{2}{3}\overrightarrow{GG'}$

Q.17 If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three unit vectors such that $3\vec{a}+4\vec{b}+5\vec{c} = \vec{0}$, then-

(A) $\vec{a} \parallel \vec{b}$ (B) $\vec{b} \parallel \vec{c}$

(C) $\vec{a} \perp \vec{b}$ (D) None of these

Q.18 If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three unit vectors such that $\vec{a}$ and $\vec{b}$ are perpendicular to each other and $2\vec{a}-3\vec{b} = \lambda\vec{c}$, then value of λ is-

(A) 1 (B) 5 (C) $\sqrt{13}$ (D) 13

Q.19 If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$, then vectors $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ will be-

(A) perpendicular (B) parallel

(C) coincident (D) None of these

Q.20 If $\vec{p}$, $\vec{q}$, $\vec{r}$ be three mutually perpendicular vectors of equal magnitude, then the angle between $\vec{p}$ and $\vec{p}+\vec{q}+\vec{r}$ is-

(A) $\cos^{-1}(1/\sqrt{3})$ (B) $\sin^{-1}(1/\sqrt{3})$

(C) $\cos^{-1}(1/3)$ (D) $\sin^{-1}(1/3)$

Q.21 If $\vec{a}, \vec{b}, \vec{c}$ are three non- coplanar vectors, then $\frac{\vec{a}.\vec{b}\times\vec{c}}{\vec{c}\times\vec{a}.\vec{b}}+\frac{\vec{b}.\vec{a}\times\vec{c}}{\vec{c}.\vec{a}\times\vec{b}}$ equals-

(A) 0 (B) 2

(C) $2\,[\vec{a}\ \vec{b}\ \vec{c}]$ (D) None of these

Q.22 If $\vec{a}$ = (1, 1 – 1), $\vec{b}$ = (1, – 1, 1), then a unit vector $\vec{c}$ which is perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$ is given by-

(A) $(1/\sqrt{3})$ (– 1, 1, 1)

(B) $(1/\sqrt{6})$ (2, 1, –1)

(C) $(1/\sqrt{6})$ (2, –1, 1)

(D) None of these

Q.23 If $\vec{a}, \vec{b}, \vec{c}$ are three non- coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined as $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ then $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$ equals-

(A) 0 (B) 1 (C) 2 (D) 3

Q.24 If $\vec{a}$, $\vec{b}$, $\vec{c}$ be any three non- zero non coplanar vectors and vectors

$\vec{p} = \frac{\vec{b}\times\vec{c}}{\vec{a}.\vec{b}\times\vec{c}}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{\vec{a}.\vec{b}\times\vec{c}}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{\vec{a}.\vec{b}\times\vec{c}}$, then

$[\vec{p}\ \vec{q}\ \vec{r}]$ equals-

(A) $\vec{a}.\vec{b}\times\vec{c}$ (B) $\frac{1}{\vec{a}.\vec{b}\times\vec{c}}$

(C) 0 (D) None of these

Q.25 Let $\vec{a}$ and $\vec{b}$ two unit vectors. If vectors $3\vec{a}-5\vec{b}$ and $\vec{a}+\vec{b}$ are perpendicular, then-

(A) $\vec{a}$ and $\vec{b}$ are perpendicular

(B) $\vec{a}$ and $\vec{b}$ are in opposite direction

(C) angle between $\vec{a}$ and $\vec{b}$ is zero

(D) None of these

Q.26 If $\vec{a} = (1, 1, 1)$, $\vec{c} = (0, 1, -1)$ are two vectors and $\vec{b}$ is a vector such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b} = 3$, then $\vec{b}$ equals-

(A) (5, 2, 2) (B) (5/3, 2/3, 2/3)

(C) (2/3, 5/3, 2/3) (D) (2/3, 2/3, 5/3)

Q.27 Let the vectors $\vec{a}$ and $\vec{b}$ are at right- angle, then what is value of m so that $\vec{a}+m\vec{b}$ and $\vec{a}+\vec{b}$ are at right angle-

(A) 1 (B) – 1

(C) 0 (D) $-(|\vec{a}|/|\vec{b}|)^2$

Q.28 $[(a\times\vec{b})\times(\vec{a}\times\vec{c})].\vec{d}$ equals-

(A) $(\vec{a}.\vec{d})[\vec{a}\ \vec{b}\ \vec{c}]$ (B) $(\vec{c}.\vec{d})[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $(\vec{b}.\vec{d})[\vec{a}\ \vec{b}\ \vec{c}]$ (D) None of these

Q.29 If $p\hat{i}+q\hat{j}+r\hat{k}$ is a unit vector and is perpendicular to vectors $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+2\hat{j}-\hat{k}$, then |p| equals-

(A) $\frac{1}{\sqrt{75}}$ (B) $\frac{2}{\sqrt{75}}$

(C) $\frac{3}{\sqrt{75}}$ (D) None of these

Q.30 If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ then -

(A) $|\vec{a}| = 1, |\vec{b}| = |\vec{c}|$

(B) $|\vec{b}| = 1, |\vec{c}| = |\vec{a}|$

(C) $|\vec{b}| = 2, |\vec{c}| = 2|\vec{a}|$

(D) $|\vec{c}| = 1, |\vec{a}| = 1$

Q.31 If vectors $\vec{c}$, $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right- handed orthogonal system, then $\vec{c}$ is-

(A) $\vec{0}$ (B) $z\hat{i}-x\hat{k}$

(C) $-z\hat{i}+x\hat{k}$ (D) $z\hat{k}$

Q.32 If $\vec{u}=\vec{a}-\vec{b}$ and $\vec{v}=\vec{a}+\vec{b}$, and $|\vec{a}| = |\vec{b}| = 2$ then $|\vec{u}\times\vec{v}|$ is equal to-

(A) $2\sqrt{16-(\vec{a}.\vec{b})^2}$ (B) $\sqrt{16-(\vec{a}.\vec{b})^2}$

(C) $2\sqrt{4-(\vec{a}.\vec{b})^2}$ (D) $\sqrt{4-(\vec{a}.\vec{b})^2}$

Q.33 If $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, where $\vec{a},\vec{b},\vec{c}$ are non- coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c}).(\vec{p}+\vec{q}+\vec{r})$ equals-

(A) 3 (B) 2

(C) 1 (D) 0

Q.34 If a and b are non- parallel unit vectors such that $|\vec{a}+\vec{b}|=\sqrt{3}$, then $(2\vec{a}-5\vec{b}).(3\vec{a}+\vec{b})$ equals-

(A) 11/2 (B) 0

(C) –11/2 (D) 13/2

Q.35 If A, B, C, D are four points in space, and $|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}| = \lambda$ (area of (ΔABC), then λ is equal to -

(A) 2 (B) 3

(C) 4 (D) 1

Q.36 If $\vec{a}.\hat{i} = 4$, then $(\vec{a}\times\hat{j}).(2\hat{j}-3\hat{k})$ equals-

(A) 0 (B) 2

(C) 12 (D) –12

Q.37 If $\vec{d} = p(\vec{a}\times\vec{b}) + q(\vec{b}\times\vec{c}) + r(\vec{c}\times\vec{a})$ and $[\vec{a}\,\vec{b}\,\vec{c}] = 1$, then $(p + q + r)$ equals-

(A) $\vec{d}.(\vec{a}+\vec{b}+\vec{c})$ (B) $\vec{a}+\vec{b}+\vec{c}$

(C) 1 (D) None of these

Q.38 Let $\vec{b} = 3\hat{j} + 4\hat{k}$, $\vec{a} = \hat{i} + \hat{j}$ and let $\vec{b}_1$ and $\vec{b}_2$ be component vectors of $\vec{b}$ parallel and perpendicular to $\vec{a}$. If $\vec{b}_1 = \frac{3}{2}\hat{i} + \frac{3}{2}\hat{j}$, then $\vec{b}_2$ is equal to-

(A) $-\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j}$

(B) $\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\vec{k}$

(C) $-\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\vec{k}$

(D) None of these

Q.39 If in a right- angled triangle ABC, the hypotenuse AB = p,

$\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$ equals-

(A) $2p^2$ (B) $p^2/2$

(C) p^2 (D) 0

Q.40 The value of x for which the angle between the vectors $\vec{a} = -3\hat{i} + x\hat{j} + \hat{k}$ and $\vec{b} = x\hat{i} + 2x\hat{j} + \hat{k}$ is acute and the angle between b and x-axis lies between π/2 and π satisfy-

(A) $x < -1$ only (B) $x > 0$

(C) $x > 1$ only (D) $x < 0$

LEVEL - 3

Q.1 If the vectors $\vec{a} = (c\log_2 x)\,\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{b} = (\log_2 x)\,\hat{i} + 2\hat{j} + (2c\log_2 x)\hat{k}$ make on obtuse angle for any $x \in (0, \infty)$, then c belongs to -

(A) $(-\infty, 0)$ (B) $(-\infty, -4/3)$

(C) $(-4/3, 0)$ (D) $(-4/3, \infty)$

Q.2 If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then -

(A) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ (B) $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$

(C) $\vec{a} + \vec{b} = \vec{c}$ (D) None of these

Q.3 Let the pairs $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{d}$ each determines a plane. Then the planes are parallel if -

(A) $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$

(B) $(\vec{a} \times \vec{c}) \cdot (\vec{b} \times \vec{d}) = 0$

(C) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

(D) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0$

Q.4 If $\vec{a}$ and $\vec{b}$ are not perpendicular to each other and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, $\vec{r} \cdot \vec{c} = 0$ then $\vec{r}$ is equal to

(A) $\vec{a} - \vec{c}$

(B) $\vec{b} + x\vec{a}$ for all scalars x

(C) $\vec{b} - \dfrac{\vec{b}.\vec{c}}{\vec{a}.\vec{c}}\vec{a}$

(D) None of these

Q.5 Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular to each other and the unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$.

If $\vec{c} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$, then-

(A) $x = \cos\theta$, $y = \sin\theta$, $z = \cos 2\theta$

(B) $x = \sin\theta$, $y = \cos\theta$, $z = \cos 2\theta$

(C) $x = y = \cos\theta$, $z^2 = \cos 2\theta$

(D) $x = y = \cos\theta$, $z^2 = -\cos 2\theta$

Q.6 If the vectors $\vec{a} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{b} = \hat{i} + b\hat{j} + b^2\hat{k}$, $\vec{c} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is-

(A) 0 (B) 1 (C) 2 (D) –1

Q.7 Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector ⊥ to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is π/6, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to -

(A) 0 (B) 1

(C) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$ (D) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

Q.8 If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, then $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$ equals-

(A) $[\vec{a}\ \vec{b}\ \vec{c}]^2$ (B) $[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $[\vec{a}\ \vec{b}\ \vec{c}]^3$ (D) None of these

Q.9 If forces of magnitudes 6 and 7 units acting in the directions $\hat{i} - 2\hat{j} + 2\hat{k}$ and $2\hat{i} - 3\hat{j} + 6\hat{k}$ respectively act on a particle which is displaced from the point P(2, –1, –3) to Q(3, –1, 1) then the work done by the forces is-

(A) 44 units (B) –4 units

(C) 7 units (D) –7 units

Q.10 If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $m = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, then

(A) $m < 0$ (B) $m > 0$

(C) $m = 0$ (D) $m = 3$.

Q.11 If the position vectors of three points A, B, C are respectively $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$ and $7\hat{i}+4\hat{j}+9\hat{k}$, then the unit vector perpendicular to the plane of triangle ABC is-

(A) $31\hat{i}-18\hat{j}-9\hat{k}$ (B) $\frac{31\hat{i}-38\hat{j}-9\hat{k}}{\sqrt{2486}}$

(C) $\frac{31\hat{i}+38\hat{j}+9\hat{k}}{\sqrt{2486}}$ (D) None of these

Q.12 Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 120^o$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, then $[(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})]^2$ is equal to-

(A) 225 (B) 275

(C) 325 (D) 300

Q.13 A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}$, $\vec{b} = \vec{\alpha} + 3\vec{\beta}$. If $|\vec{\alpha}| = |\vec{\beta}| = 2$ and the angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the length of a diagonal of the parallelogram is-

(A) $4\sqrt{5}$ (B) $\sqrt{3}$

(C) $4\sqrt{7}$ (D) None of these

Q.14 $(\vec{a} + 2\vec{b} - \vec{c}) \cdot \{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})\}$ is equal to

(A) $[\vec{a}\ \vec{b}\ \vec{c}]$ (B) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $3[\vec{a}\ \vec{b}\ \vec{c}]$ (D) 0

Q.15 If the vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular, then $\vec{a} \times \{\vec{a} \times \{\vec{a} \times (\vec{a} \times \vec{b})\}\}$ is equal to-

(A) $|\vec{a}|^2\vec{b}$ (B) $|\vec{a}|^3\vec{b}$

(C) $|\vec{a}|^4\vec{b}$ (D) None of these

Q.16 The area of parallelogram constructed on the vectors $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$ where $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 30^o is-

(A) 3/2 (B) 1

(C) 0 (D) None of these

➤ Statement type Questions

Each of the questions (Q.No.17 to 27) given below consists of Statement -I and Statement-II. Use the following key to choose the appropriate answer.

(A) If both Statement- I Statement- II are true, and Statement-II is the correct explanation of Statement- I.

(B) If Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I

(C) If Statement-I is true but Statement-II is false

(D) If Statement-I is false but Statement-II is true

Q. 17 **Statement-1 (A) :** If the difference of two unit vectors is again a unit vector then angle between them is 60^o

Statement-2 (R) : If angle between $\vec{a}$ & $\vec{b}$ is acute than $|\vec{a}.\vec{b}| < |\vec{a}|\,|\vec{b}|$

Q.18 **Statement-1 (A) :** ABCDEF is a regular hexagon and $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{CD} = \vec{c}$, then $\overrightarrow{EA}$ is equal to $-(\vec{b} + \vec{c})$.

Statement-2 (R) : $\overrightarrow{AE} = \overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$

Q.19 **Statement-1(A) :**In ABC, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

Statement-2 (R) : If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ then $\overrightarrow{AB} = \vec{a} + \vec{b}$ (Triangle law of addition)

Q.20 **Statement-1 (A) :** $\vec{a} = \hat{i} + p\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + q\hat{k}$ are parallel vector. If p =3/2, q = 4.

Statement-2 (R) : If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Q.21 **Statement-1 (A) :** If $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors then the vectors $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are also coplanar.

Statement-2 (R) : If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar vectors then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

Q.22 **Statement-1 (A) :** Three points A($\vec{a}$), B($\vec{b}$), C($\vec{c}$) are collinear if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Statement-2 (R) : Points $\vec{A}$, $\vec{B}$, $\vec{C}$ are collinear $\Leftrightarrow$ $\overrightarrow{AB} = t\ \overrightarrow{AC}$, $t \in R$.

Q.23 Let $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$, $\overrightarrow{UP}$ denote the sides of a regular hexagon.

Statement-1 (A) : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$

Statement-2 (R) : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} = \vec{0}$

Q.24 **Statement-1 (A) :**

Vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ & $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar for only two values of λ.

Statement-2 (R) : Three vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$.

Q.25 **Statement-1 (A) :** Three vector $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar then $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ are also non coplanar.

Statement-2 (R) : $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}]$

$= [\vec{a}\ \vec{b}\ \vec{c}]$

Q.26 **Statement-1(A) :** If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors then vectors

$2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c}$,

$\vec{a} + \vec{b} - 3\vec{c}$ are also non coplanar.

Statement-2 (R) : Three vector $\vec{A}$, $\vec{B}$, $\vec{C}$ are non coplanar then $[\vec{A}\ \vec{B}\ \vec{C}] \neq 0$

Q.27 **Statement-1 (A) :** If $\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors then $[2\vec{a} - \vec{b}\ \ 2\vec{b} - \vec{c}\ \ 2\vec{c} - \vec{a}] = 0$

Statement-2 (R) : $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

➤ Passage Based Question

Passage-1

The scalar triple product of three vectors $\vec{a}, \vec{b}$, $\vec{c}$ is denoted by $[\vec{a}\ \vec{b}\ \vec{c}]$ and is defined as $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$. Three vectors $\vec{a}, \vec{b}$, $\vec{c}$ are coplanar vectors if and only if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$. Volume of the parallelopiped whose three concurrent edges are $\vec{a}, \vec{b}$, $\vec{c}$ is $|[\vec{a}\ \vec{b}\ \vec{c}]|$

Q.28 If the volume of a parallelopiped whose three concurrent edges are $-12\hat{i} + \lambda\hat{k}$, $3\hat{j} - \hat{k}$ and $2\hat{i} + \hat{j} - 15\hat{k}$ is 546 then λ =

(A) 2/3 (B) –1

(C) –4 (D) –3

Q.29 If $\vec{a}, \vec{b}, \vec{c}$, $\vec{d}$ are four coplanar points then $[\vec{b}\ \vec{c}\ \vec{d}] + [\vec{c}\ \vec{a}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{d}]$ is

(A) 0 (B) 1

(C) $[\vec{a}\ \vec{b}\ \vec{c}]$ (D) $2[\vec{a}\ \vec{b}\ \vec{c}]$

Passage - 2 :

Let $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and let the equations $\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{b}' = \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{c}' = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ are reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$.

On the basis of above information, answer the following questions.

Q.30 The value of the expression $\vec{a} \cdot \vec{a}' + \vec{b} \cdot \vec{b}' + \vec{c} \cdot \vec{c}'$ equals-

(A) 0 (B) 1

(C) 2 (D) 3

Q.31 The expression $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is

(A) a unit vector

(B) null vector

(C) $\dfrac{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2}{|\vec{a}'|^2 + |\vec{b}'|^2 + |\vec{c}'|^2}$

(D) arbitrary vector

Q.32 The value of the expression

$\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}'$ is-

(A) $\dfrac{\vec{a} + \vec{b} - \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ (B) $\dfrac{\vec{a} - \vec{b} + \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

(C) $\dfrac{-\vec{a} + \vec{b} + \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ (D) $\dfrac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

Q.33 If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$,

$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$, then $\vec{c}' \times \vec{a}'$ equals

(A) $\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{3}$ (B) $\dfrac{\hat{i} - \hat{j} - 2\hat{k}}{3}$

(C) $\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{9}$ (D) $\dfrac{-\hat{i} + \hat{j} - 2\hat{k}}{3}$

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If $\vec{a}, \vec{b}, \vec{c}$ are three non zero vectors out of which two are not collinear. If $\vec{a} + 2\vec{b}$ & $\vec{c}$; $\vec{b} + 3\vec{c}$ and $\vec{a}$ are collinear then $\vec{a} + 2\vec{b} + 6\vec{c}$ is – **[AIEEE- 2002]**

(A) Parallel to $\vec{c}$ (B) Parallel to $\vec{a}$

(C) Parallel to $\vec{b}$ (D) $\vec{0}$

Q.2 If $[\vec{a}\ \vec{b}\ \vec{c}] = 4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] =$ **[AIEEE- 2002]**

(A) 4 (B) 2 (C) 8 (D) 16

Q.3 If $\vec{c} = 2\lambda\ (\vec{a}\times\vec{b}) + 3\mu\ (\vec{b}\times\vec{a})$; $\vec{a}\times\vec{b} \neq \vec{0}$, $\vec{c}\,.\,(\vec{a}\times\vec{b}) = 0$ then- **[AIEEE-2002]**

(A) $\lambda = 3\mu$ (B) $2\lambda = 3\mu$

(C) $\lambda + \mu = 0$ (D) None of these

Q.4 If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$, then along Component of $\vec{a}$ on $\vec{b}$ is- **[AIEEE-2002]**

(A) $3\hat{i} - 3\hat{j} + \hat{k}$ (B) $\dfrac{9(5\hat{i} - 3\hat{j} + \hat{k})}{35}$

(C) $\dfrac{(5\hat{i} - 3\hat{j} + \hat{k})}{35}$ (D) $9\ (5\hat{i} - 3\hat{j} + \hat{k})$

Q.5 A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is- **[AIEEE- 2002]**

(A) $\dfrac{4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$ (B) $\dfrac{2\hat{i} - 6\hat{j} - 3\hat{k}}{7}$

(C) $\dfrac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$ (D) $\dfrac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$

Q.6 Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\,.\,\hat{n} = 0$ and $\vec{v}\,.\,\hat{n} = 0$, then $|\vec{w}\,.\,\hat{n}|$ is equal to- **[AIEEE- 2003]**

(A) 3 (B) 0 (C) 1 (D) 2

Q.7 A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is- **[AIEEE-2003]**

(A) 50 units (B) 20 units

(C) 30 units (D) 40 units

Q.8 The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ & $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is- **[AIEEE-2003]**

(A) $\sqrt{288}$ (B) $\sqrt{18}$ (C) $\sqrt{72}$ (D) $\sqrt{33}$

Q.9 $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$, then $(\vec{a}\,.\,\vec{b} + \vec{b}\,.\,\vec{c} + \vec{c}\,.\,\vec{a})$ is equal to- **[AIEEE- 2003]**

(A) 1 (B) 0 (C) – 7 (D) 7

Q.10 Consider points A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then ABCD is a- **[AIEEE- 2003]**

(A) parallelogram but not a rhombus

(B) square

(C) rhombus

(D) None of these

Q.11 If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non- coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w})\,.\,(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})$ equals **[AIEEE- 2003]**

(A) $3\,\vec{u}\,.\,\vec{v}\times\vec{w}$ (B) 0

(C) $\vec{u}\,.\,\vec{v}\times\vec{w}$ (D) $\vec{u}\,.\,\vec{w}\times\vec{v}$

Q.12 If $\vec{a}, \vec{b}, \vec{c}$ are non- coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non- coplanar for **[AIEEE- 2004]**

(A) all values of λ

(B) all except one value of λ

(C) all except two values of λ

(D) no value of λ

Q.13 Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$, $\vec{v}$ & $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals- **[AIEEE- 2004]**

(A) 2 (B) $\sqrt{7}$

(C) $\sqrt{14}$ (D) 14

Q.14 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non- zero vectors such that no two are collinear and

$(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$ If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then sin θ equals- **[AIEEE- 2004]**

(A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{2\sqrt{2}}{3}$

Q.15 For any vector $\vec{a}$,

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is equal to – **[AIEEE- 2005]**

(A) $|\vec{a}|^2$ (B) $2|\vec{a}|^2$

(C) $3|\vec{a}|^2$ (D) None of these

Q.16 If C is the mid point of AB and P is any point outside AB, then - **[AIEEE-2005]**

(A) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

(B) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(C) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$

(D) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

Q.17 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number then

$[\lambda(\vec{a} + \vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}] = [\vec{a}\ \ \vec{b} + \vec{c}\ \ \vec{b}]$ for - **[AIEEE-2005]**

(A) exactly one value of λ

(B) no value of λ

(C) exactly three values of λ

(D) exactly two values of λ

Q.18 If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, where $\vec{a}$, $\vec{b}$ & $\vec{c}$ are any three vectors such that $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are – **[AIEEE-2006]**

(A) inclined at an angle of π/6 between them

(B) perpendicular

(C) parallel

(D) inclined at an angle of π/3 between them

Q.19 ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is– **[AIEEE- 2006]**

(A) $\frac{(AB)(AC)}{AB + AC}$ (B) $\frac{1}{AB} + \frac{1}{AC}$

(C) $\frac{1}{AD}$ (D) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$

Q.20 The values of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right-angled triangle with C = $\frac{\pi}{2}$ are – **[AIEEE- 2006]**

(A) –2 and –1 (B) –2 and 1

(C) 2 and –1 (D) 2 and 1

Q.21 If $\vec{u}$ and $\vec{v}$ are unit vectors and θ is the acute angle between them, then $2\vec{u} \times 3\vec{v}$ is a unit vector for – **[AIEEE- 2007]**

(A) exactly two values of θ

(B) more than two values of θ

(C) no value of θ

(D) exactly one value of θ

Q.22 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ & $\vec{b}$ then x equals - **[AIEEE- 2007]**

(A) 0 (B) 1 (C) –4 (D) –2

Q.23 The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[AIEEE- 2008]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 0

Q.24 The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ & $\vec{c} = \hat{j} + \hat{k}$ & bisects the angle between $\vec{b}$ & $\vec{c}$. Then which one of the following gives possible values of α & β ? **[AIEEE- 2008]**

(A) α = 1, β = 2 (B) α = 2, β = 1

(C) α = 1, β = 1 (D) α = 2, β = 2

Q.25 If $\vec{u},\vec{v},\vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{w}]-[p\vec{v}\ \vec{w}\ q\vec{u}]-[2\vec{w}\ q\vec{v}\ q\vec{u}]=0$ holds for : **[AIEEE -2009]**
(A) exactly two values of (p, q)
(B) more than two but not all values of (p, q)
(C) all values of (p, q)
(D) exactly one value of (p, q)

Q.26 Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}=3$ is – **[AIEEE -2010]**
(A) $-\hat{i}+\hat{j}-2\hat{k}$ (B) $2\hat{i}-\hat{j}+2\hat{k}$
(C) $\hat{i}-\hat{j}-2\hat{k}$ (D) $\hat{i}+\hat{j}-2\hat{k}$

Q.27 If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) = **[AIEEE -2010]**
(A) (–3, 2) (B) (2, –3)
(C) (–2, 3) (D) (3, –2)

Q.28 If $\vec{a}=\frac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$ and $\vec{b}=\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $(2\vec{a}-\vec{b})\bullet[(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})]$ is - **[AIEEE -2011]**
(A) –5 (B) –3 (C) 5 (D) 3

Q.29 The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to - **[AIEEE -2011]**
(A) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (B) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$
(C) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (D) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Q.30 Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[AIEEE -2012]**
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.31 Let ABCD be a parallelogram such that $\overrightarrow{AB}=\vec{q}$, $\overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE -2012]**
(A) $\vec{r}=-\vec{q}+\left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$
(B) $\vec{r}=\vec{q}-\left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$
(C) $\vec{r}=-3\vec{q}+\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$
(D) $\vec{r}=3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

Q.32 If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is – **[JEE Main - 2013]**
(A) $\sqrt{33}$ (B) $\sqrt{45}$
(C) $\sqrt{18}$ (D) $\sqrt{72}$

SECTION-B

Q.1 Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ & $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is – **[IIT- 1993/ AIEEE -2005]**
(A) The Arithmetic mean of a and b
(B) The Geometric mean of a and b
(C) The Harmonic mean of a and b
(D) Equal to zero+

Q.2 Let α,β,γ be distinct real numbers. The points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$, $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$ - **[IIT Scr. 1994]**
(A) are collinear
(B) form an equilateral triangle
(C) form an isosceles triangle
(D) form a right angled triangle

Q.3 Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{j}-\hat{k}$, $\vec{c}=\hat{k}-\hat{i}$. If $\hat{d}$ is a unit vector such that $\vec{a}\cdot\hat{d}=0=[\vec{b}\ \vec{c}\ \hat{d}]$, then $\hat{d}$ equals- **[IIT -1995]**
(A) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ (B) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$
(C) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (D) $\pm\hat{k}$

Q.4 If $\vec{a}, \vec{b}, \vec{c}$ are non- coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is- **[IIT- 1995]**

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Q.5 A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. The system is rotated thro' a certain angle about the origin in the counterclockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and 1, then **[IIT- 1996]**

(A) p = 0 (B) p = 1 or p = $-\frac{1}{3}$

(C) p = – 1 or p = $\frac{1}{3}$ (D) p = 1 or p = – 1

Q.6 Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 10\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where O, A, C are non- collinear. Let p denote the area of the quadrilateral OABC and q denote the area of the parallelogram with OA and OC as adjacent sides. Then $\frac{p}{q}$ is equal to- **[IIT- 1997]**

(A) 4 (B) 6

(C) $\frac{1}{2}\frac{|\vec{a} - \vec{b}|}{|\vec{a}|}$ (D) None of these

Q.7 If $\vec{a}, \vec{b}$ & $\vec{c}$ are vectors such that $|\vec{b}| = |\vec{c}|$, then $[(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})] \times (\vec{b} \times \vec{c}).(\vec{b} + \vec{c}) =$ **[IIT- 1997]**

(A) 1 (B) – 1

(C) 0 (D) None of these

Q.8 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) - \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is- **[IIT- 1997]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) None of these

Q.9 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then- **[IIT- 1998]**

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$

(C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

Q.10 Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is vector such that $\vec{a}.\vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30º then $|(\vec{a} \times \vec{b}) \times \vec{c}| =$ **[IIT- 1999]**

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) 2 (D) 3

Q.11 Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of three vertices. A, B, C of a triangle respectively. Then the area of this triangle is given by- **[IIT -2000]**

(A) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

(B) $\frac{1}{2}(\vec{a} \times \vec{b}).\vec{c}$

(C) $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

(D) None of these

Q.12 Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on - **[IIT scr. 2001/AIEEE -2005]**

(A) only x (B) only y

(C) neither x nor y (D) both x and y

Q.13 If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed- **[IIT- 2001]**

(A) 4 (B) 9 (C) 8 (D) 6

Q.14 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is **[IIT scr. 2002]**

(A) 45º (B) 60º

(C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

Q.15 Let $\vec{V} = 2\hat{i}+\hat{j}-\hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector; then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is – **[IIT scr. 2002]**

(A) – 1 (B) $\sqrt{10}+\sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

Q.16 If $\vec{a} = \hat{i}+a\hat{j}+\hat{k}$; $\vec{b} = \hat{j}+a\hat{k}$; $\vec{c} = a\hat{i}+\hat{k}$, then find the value of 'a' for which volume of parallelopiped formed by these three vectors as coterminous edges, is minimum. **[IIT Scr.2003]**

(A) $\sqrt{3}$ (B) 3 (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{3}$

Q.17 If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ & $\vec{a}.\vec{b} = 1$ & $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$ then $\vec{b}$ is equal to- **[IIT Scr.2004]**

(A) $2\hat{i}$ (B) $\hat{i}-\hat{j}+\hat{k}$

(C) $\hat{i}$ (D) $2\hat{j}-\hat{k}$

Q.18 A unit vector is orthogonal to $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar to $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ then the vector, is- **[IIT Scr.2004]**

(A) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (B) $\frac{2\hat{i}+5\hat{j}}{\sqrt{29}}$

(C) $\frac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (D) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

Q.19 Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$ & $\vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is of length $\frac{1}{\sqrt{3}}$ unit is – **[IIT-2006]**

(A) $4\hat{i}+\hat{j}-4\hat{k}$ (B) $4\hat{i}-\hat{j}+4\hat{k}$

(C) $2\hat{i}+\hat{j}-2\hat{k}$ (D) $3\hat{i}+\hat{j}-3\hat{k}$

Q.20 The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is - **[IIT-2007]**

(A) zero (B) one (C) two (D) three

Q.21 Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. Which one of the following is correct ? **[IIT-2007]**

(A) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} = \vec{0}$

(B) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq \vec{0}$

(C) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{a}\times\vec{c} \neq \vec{0}$

(D) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ are mutually perpendicular

Q.22 The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = 1/2$. Then, the volume of the parallelopiped is **[IIT-2008]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Q.23 If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that - $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$, then - **[IIT-2009]**

(A) $\vec{a},\vec{b},\vec{c}$ are non-coplanar

(B) $\vec{b},\vec{c},\vec{d}$ are non-coplanar

(C) $\vec{b},\vec{d}$ are non-parallel

(D) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

Q.24 Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a - **[IIT-2010]**

(A) Parallelogram, which is neither a rhombus nor a rectangle

(B) Square

(C) Rectangle, but not a square

(D) Rhombus, but not a square

Q.25 If $\vec{a}$ and $\vec{b}$ are vector is space given by $\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is – **[IIT-2010]**

(A) – 5 (B) 5

(C) 4 (D) none of these

Q.26 Two adjacent sides of a parallelogram ABCD are given by $\vec{AB} = 2\hat{i}+10\hat{j}+11\hat{k}$ and $\vec{AD} = -\hat{i}+2\hat{j}+2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by - **[IIT-2010]**

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Q.27 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by - **[IIT-2011]**

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$
(C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Q.28 The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is /are - **[IIT-2011]**

(A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$
(C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

Q.29 Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is - **[IIT-2011]**

(A) 6 (B) 7 (C) 8 (D) 9

Q.30 If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is - **[IIT-2011]**

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) π

Q.31 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is **[IIT-2012]**

Q.32 If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is **[IIT-2012]**

(A) 0 (B) 3 (C) 4 (D) 8

Q.33 Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is – **[JEE - Advance 2013]**

(A) 5 (B) 20 (C) 10 (D) 30

Q.34 Match List-I with List-II and select the correct answer using the code given below the lists : **[JEE - Advance 2013]**

	List – I	List – II
(P)	Volume of parallelepiped determined by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b})$, $3(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ is	(1) 100
(Q)	Volume of parallelepiped determined by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b})$, $(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(2) 30
(R)	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is	(3) 24
(S)	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(4) 60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

Q.35 Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE - Advance 2013]**

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	D	D	B	D	B	C	C	D	A	A	D	D	C	A	A	A	A	A
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	C	C	B	B	C	B	C	D	B	B	C	D	A	C	B	B	C	B
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	B	A	B	A	D	C	C	C	A	A	D	A	D	C	B	B	A	A	A	B
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	A	A	D	B	D	B	A	B	B	A	C	D	B	C	C	D	B	B	A	B
Q.No.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	A	B	A	B	C	D	D	D	B	D	B	B	B	A	A	C	B	C	B	B
Q.No.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	D	A	B,C	A	D	C	D	A	D	D	B	D	A	D	C	B	C	A	C	B
Q.No.	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
Ans.	D	B	B	B	C	D	A	D	C	A	A	D	C	C	B	D	A	B	D	B
Q.No.	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
Ans.	C	C	B	A	D	D	C	B,C	C	D	C	D	A	B	B	A	C	D	B	D
Q.No.	161	162	163																	
Ans.	D	D	B																	

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	A	A	A	A	B	A	A	D	A	B	B	D	B	C	C	C	B	A
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	C	D	B	B	B	D	A	A	B	B	A	A	C	C	D	A	C	C	D

LEVEL- 3

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	C	C	D	D	C	A	A	A	B	D	C	C	C	A	B	A	C	A
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33							
Ans.	A	A	C	A	C	A	A	D	C	D	B	D	B							

LEVEL- 4

SECTION-A

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	D	B	B	C	A	D	D	C	D	C	C	C	D	B	A	B	C	C	D
Qus.	21	22	23	24	25	26	27	28	29	30	31	32								
Ans.	D	D	C	C	D	A	A	A	D	B	A	A								

SECTION-B

1.[B] vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$– ac – a(b – c) + c(c) = 0$

$c^2 = ab$

c is G.M. of a and b.

2.[B] $\overrightarrow{AB} = (\beta-\alpha)\hat{i} + (\gamma-\beta)\hat{j} + (\alpha-\gamma)\hat{k}$

$|\overrightarrow{AB}| = \sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$

$\overrightarrow{BC} = (\gamma-\beta)\hat{i} + (\alpha-\gamma)\hat{j} + (\beta-\alpha)\hat{k}$

$|\overrightarrow{BC}| = \sqrt{(\gamma-\beta)^2+(\alpha-\gamma)^2+(\beta-\alpha)^2}$

$\overrightarrow{CA} = (\alpha-\gamma)\hat{i} + (\beta-\alpha)\hat{j} + (\gamma-\beta)\hat{k}$

$|\overrightarrow{CA}| = \sqrt{(\alpha-\gamma)^2+(\beta-\alpha)^2+(\gamma-\beta)^2}$

$|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}|$

3.[A] $\vec{a}\cdot\vec{d} = 0$ $\quad [\vec{b}\ \vec{c}\ \vec{d}] = 0$

$\vec{d} = (\vec{b}\times\vec{c})\times\vec{a}$

$\vec{d} = (\vec{a}\,.\,\vec{b})\vec{c} - (\vec{a}\,.\,\vec{c})\vec{b}$

$\vec{d} = (-1)\vec{c} - (-1)\vec{b}$

$\vec{d} = \vec{b} - \vec{c} = \hat{i}+\hat{j}-2\hat{k}$

$\hat{d} = \pm\dfrac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$

Aliter

Let $\vec{d} = x\hat{i}+y\hat{j}+2\hat{k}$

$|\vec{d}| = 1$

$\Rightarrow x^2+y^2+z^2 = 1 \qquad \dots (1)$

$\vec{a}\,.\,\vec{d} = 0$

$\Rightarrow x - y = 0 \qquad \dots (2)$

$[\vec{b}\ \vec{c}\ \vec{d}] = 0$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y + z = 0 \qquad \dots (3)$

Solving (1), (2) & (3)

$\vec{d} = \pm\dfrac{(\hat{i}+\hat{j}-2\hat{k})}{\sqrt{6}}$

4.[A] $\vec{a}\times(\vec{b}\times\vec{c}) = \dfrac{\vec{b}+\vec{c}}{\sqrt{2}}$

$(\vec{a}\,.\,\vec{c})\vec{b} - (\vec{a}\,.\,\vec{b})\vec{c} = \dfrac{\vec{b}+\vec{c}}{\sqrt{2}}$

$\vec{a}\,.\,\vec{c} = \dfrac{1}{\sqrt{2}}$ $\qquad \vec{a}\,.\,\vec{b} = -\dfrac{1}{\sqrt{2}}$

$|\vec{a}||\vec{c}|\cos\theta = \dfrac{1}{\sqrt{2}}$ $\qquad |\vec{a}||\vec{b}|\cos\phi = -\dfrac{1}{\sqrt{2}}$

$\cos\theta = \dfrac{1}{\sqrt{2}}$ $\qquad \cos\phi = -\dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$ $\qquad \theta = \dfrac{3\pi}{4}$

angle b/w $\vec{a}$ & $\vec{c}$ $\qquad$ angle b/w $\vec{a}$ & $\vec{b}$

5.[B] Magnitude will remain same

$\sqrt{(2p)^2+(1)^2} = \sqrt{(p+1)^2+(1)^2}$

$(2p)^2 = (p+1)^2$

$\pm 2p = p + 1$

$p = 1, -\dfrac{1}{3}$

6.[B] p = area of quadrilateral OABC = $\dfrac{1}{2}|\overrightarrow{OB}\times\overrightarrow{AC}|$

$= \dfrac{1}{2}|(10\vec{a}+2\vec{b})\times(\vec{b}-\vec{a})|$

$= \dfrac{1}{2}|10(\vec{a}\times\vec{b}) - 2(\vec{b}\times\vec{a})|$

$= \dfrac{1}{2}|12(\vec{a}\times\vec{b})|$

$p = 6|\vec{a}\times\vec{b}|$

$p = 6q$

$\dfrac{p}{q} = 6$

7.[C] $[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]\times(\vec{b}\times\vec{c}).(\vec{b}+\vec{c})$

$= [\vec{a}\times\vec{c} + \vec{b}\times\vec{a} + \vec{b}\times\vec{c}]\times(\vec{b}\times\vec{c}).(\vec{b}+\vec{c})$

$= [(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{a})\times(\vec{b}\times\vec{c})].(\vec{b}+\vec{c})$

$= [-\{(\vec{a}\times\vec{c}).\vec{b}\}\vec{c} + (\vec{b}\times\vec{a}).\vec{c}\}\vec{b}].(\vec{b}+\vec{c})$

$= \{-[\vec{a}\ \vec{c}\ \vec{b}]\vec{c} + [\vec{b}\ \vec{a}\ \vec{c}]\vec{b}\}.(\vec{b}+\vec{c})$

$= \{+[\vec{a}\ \vec{b}\ \vec{c}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{b}\}.(\vec{b}+\vec{c})$

$= [\vec{a}\ \vec{b}\ \vec{c}](\vec{c}-\vec{b}).(\vec{b}+\vec{c})$

$= [\vec{a}\ \vec{b}\ \vec{c}][|\vec{c}|^2 - |\vec{b}|^2]$

$= 0 \qquad (\because |\vec{b}| = |\vec{c}|)$

8.[B] $|\vec{a}| = |\vec{b}| = 1,\ |\vec{c}| = 2$

$\vec{a} \times (\vec{a} \times \vec{c}) = \vec{b}$

$|\vec{a}||\vec{a} \times \vec{c}| \sin 90^o = 1$

$1.|\vec{a}||\vec{c}| \sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}$

9.[D] $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent

$\therefore \vec{a}\ \vec{b}\ \vec{c}$ are coplanar

$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$\beta = 1$

$|\vec{c}| = 3$

$1 + \alpha^2 + \beta^2 = 3$

$\alpha = \pm 1 \quad (\because \beta = 1)$

10.[B] $|(\vec{a} \times \vec{b}) \times \vec{c}|$

$= |\vec{a} \times \vec{b}||\vec{c}| \sin 30^o$

$= \frac{|\vec{a} \times \vec{b}||\vec{c}|}{2}$

$= 3.1\frac{1}{2} = \frac{3}{2}$

$|\vec{c} - \vec{a}| = 2\sqrt{2}$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a} = 8$

$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$

$(|\vec{c}| - 1)^2 = 0$

$|\vec{c}| = 1$

$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

11.[C] If $\vec{a}, \vec{b}, \vec{c}$ Position vector of three vertices of ΔABC

Then area of $\Delta = \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

12.[C] $$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= 1.(1 + x - y - x + x^2) - 1(x^2 - y)$

$= 1$

$\therefore [\vec{a}\ \vec{b}\ \vec{c}]$ depends neither x nor y

13.[B] $|\vec{a} + \vec{b} + \vec{c}| \geq 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \geq 0$

$2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \geq -3$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} \geq -\frac{3}{2}$

$-2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \leq 3$

$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$

$= 2\{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\} - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$\leq 2(3) + 3$

≤ 9

14.[B] $|\vec{a}| = |\vec{b}| = 1$

$\vec{a} + 2\vec{b} \perp 5\vec{a} - 4\vec{b}$

$(\vec{a} + 2\vec{b}).(5\vec{a} - 4\vec{b}) = 0$

$5|\vec{a}|^2 + 6\vec{a}.\vec{b} - 8|\vec{b}|^2 = 0$

$5(1) + 6|\vec{a}||\vec{b}| \cos\theta - 8 = 0$

$\cos\theta = 1/2$

$\theta = 60^o$

15.[C] $[\vec{U}\ \vec{V}\ \vec{W}] = \vec{U}.(\vec{V} \times \vec{W})$

$= |\vec{U}||\vec{V} \times \vec{W}| \cos\theta$

$= |\vec{V} \times \vec{W}| \cos\theta$

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3\hat{i} - 7\hat{j} - \hat{k}$$

$|\vec{V} \times \vec{W}| = \sqrt{9+49+1} = \sqrt{59}$

$[\vec{U}\ \vec{V}\ \vec{W}] = \sqrt{59}\ \cos\theta$

max of $[\vec{U}\ \vec{V}\ \vec{W}] = \sqrt{59}$ $(\because \cos\theta = 1)$

16.[C] V = Volume of parallelopiped = $[\vec{a}\ \vec{b}\ \vec{c}]$

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = a^3 - a + 1$$

for max. or min. $V' = 3a^2 - 1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

$V'' = 6a$

$V'' = +ve$ if $a = \frac{1}{\sqrt{3}}$

V is min. if $a = \frac{1}{\sqrt{3}}$

17.[C] $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ we take $\vec{b}$ by option s.t $\vec{a}.\vec{b} = 1$

& $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

18.[A] Let the required unit vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$a_1^2 + a_2^2 + a_3^2 = 1$ ...(1)

$\vec{a}$ is orthogonal to $3\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore 3a_1 + 2a_2 + 6a_3 = 0$ ...(2)

$\vec{a}$, $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ are coplanar

$$\therefore \begin{vmatrix} a_1 & a_2 & a_3 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$2a_1 - a_2 - 3a_3 = 0$ ...(3)

from (2) and (3)

$a_1 = 0$, $a_2 = -3a_3$, Put in equation (1)

$9a_3^2 + a_3^2 = 1 \Rightarrow a_3 = \pm \frac{1}{\sqrt{10}}$

$a_1 = 0$, $a_2 = \mp \frac{3}{\sqrt{10}}$, $a_3 = \pm \frac{1}{\sqrt{10}}$

$$\vec{a} = \pm \left(\frac{3}{\sqrt{10}}\hat{j} - \frac{\hat{k}}{\sqrt{10}} \right)$$

19.[B] $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = \hat{i} + \hat{j} - \hat{k}$

Let $\vec{d}$ is a vector lie in plane of $\vec{a}$ and $\vec{b}$ therefore $\vec{d}$ can be written as

$\vec{d} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{d} = (\lambda + 1)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$

Projection of $\vec{d}$ on $\vec{c} = \frac{\vec{d}.\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$

$$\frac{(\lambda+1)+(2-\lambda)-(\lambda+1)}{\sqrt{3}} = \pm \frac{1}{\sqrt{3}}$$

taking (– ive sign) we get $\lambda = 3$

Required vector $\vec{d} = 4\hat{i} - \hat{j} + 4\hat{k}$

Aliter

Let $\vec{d}$ be the required vector

$\because$ Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \vec{d}$ is coplanar with $\vec{a}$ & $\vec{b}$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3x - 3z = 0$

$\Rightarrow x = z$ …(1)

Projection of $\vec{d}$ on $\vec{c}$

$$\left| \frac{\vec{d}.\vec{c}}{|\vec{c}|} \right| = \frac{1}{\sqrt{3}}$$

$$\frac{x+y-z}{\sqrt{3}} = \pm \frac{1}{\sqrt{3}}$$

$x + y - z = \pm 1$

$\Rightarrow y = \pm 1$ … (2)

Now check the options.

20.[C] given vector are coplanar $\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 2-\lambda^2 & 1 & 1 \\ 2-\lambda^2 & -\lambda^2 & 1 \\ 2-\lambda^2 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\Rightarrow (2-\lambda^2)\begin{vmatrix} 0 & 0 & 1+\lambda^2 \\ 0 & -\lambda^2-1 & 1+\lambda^2 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$(2 - \lambda^2)(1 + \lambda^2)^2 = 0 \quad \lambda = \pm\sqrt{2}$

21.[B] $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\vec{a} \times \vec{b} = (\vec{c} \times \vec{a})$ ...(1)

$\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{b} \times \vec{a} + \vec{b} \times \vec{c} = 0$

$\vec{b} \times \vec{c} = \vec{a} \times \vec{b}$ ...(2)

$\vec{c} \times (\vec{a} + \vec{b} + \vec{c}) = 0$

$\vec{c} \times \vec{a} + \vec{c} \times \vec{b} = 0$

$\vec{c} \times \vec{a} = \vec{b} \times \vec{c}$ ...(3)

From (1), (2), (3) we get

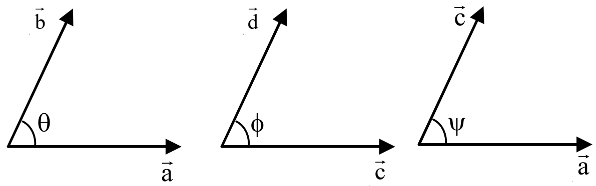
$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

22.[A] Volume of parallelopiped = $[\hat{a}\ \hat{b}\ \hat{c}]$

$$= \sqrt{\begin{vmatrix} \hat{a}.\hat{a} & \hat{a}.\hat{b} & \hat{a}.\hat{c} \\ \hat{b}.\hat{a} & \hat{b}.\hat{b} & \hat{b}.\hat{c} \\ \hat{c}.\hat{a} & \hat{c}.\hat{b} & \hat{c}.\hat{c} \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}$$

23.[C] $|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{d}| = 1$



$\vec{a} \times \vec{b} = \sin\theta\ \hat{n}_1$ $\qquad$ $\vec{a}\,.\,\vec{c} = 1/2$

$\vec{c} \times \vec{d} = \sin\phi\ \hat{n}_2$ $\qquad$ $\cos\psi = 1/2$

$(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = 1$ $\qquad$ $\psi = 60^o$

$\sin\theta\,\hat{n}_1 .\ \sin\phi\ \hat{n}_2 = 1$

$\sin\theta \sin\phi\ \hat{n}_1 . \hat{n}_2 = 1$

$\sin\theta \sin\phi \cos\alpha = 1$ ...(1)

where α is angle between $\hat{n}_1$ and $\hat{n}_2$

equation (1) is satisfies if $\theta = \phi = \pi/2$, $\alpha = 0$

$\psi = 60^o$

above result show that $\vec{b}$ and $\vec{d}$ are non parallel.

24.[A] $\overrightarrow{PQ} = 6i + j$; $\overrightarrow{RS} = 6i + j$

$\overrightarrow{RQ} = i - 3j$; $\overrightarrow{SP} = i - 3j$

$\left|\overrightarrow{PQ}\right| \neq \left|\overrightarrow{RQ}\right|$ ($\therefore$ not a rhombus or a rectangle)

PQ || RS; RQ || SP

Also $\overrightarrow{PQ}\,.\,\overrightarrow{RQ} \neq 0$

$\therefore$ PQRS is not a square

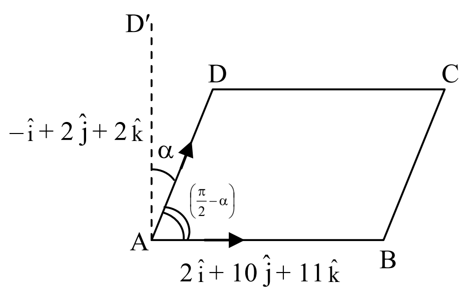
$\Rightarrow$ PQRS is a parallelogram

25.[5] $|\bar{a}| = |\bar{b}| = 1 \ \&\ \bar{a}\cdot\bar{b} = 0$

$(2\bar{a} + \bar{b})\cdot[(\bar{a}\times\bar{b})\times(\bar{a} - 2\bar{b})]$

$= (2\bar{a} + \bar{b})\cdot[\bar{b} + 2\bar{a}] = |\bar{b}|^2 + 4|\bar{a}|^2 = 5$

26.[B]



$$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \frac{40}{3(15)} = \frac{8}{9}$$

$$\sin\alpha = \frac{8}{9} \Rightarrow \cos\alpha = \frac{\sqrt{17}}{9}$$

27.[C] Let $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$

$\because\ [\vec{a}\ \ \vec{b}\ \ \vec{v}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = 0$$

On solving $x = z$ ....(1)

$\because$ projection of $\vec{v}$ on $\vec{c}$ is $\frac{1}{\sqrt{3}}$

So, $\frac{1}{\sqrt{3}} = \frac{\vec{v}\,.\,\vec{c}}{|\vec{c}|} \Rightarrow \frac{x - y - z}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow\ x - y - z = 1$ ....(2)

So solving (1) & (2)

$y = -1$ & $x = z$

28.[A, D] $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is coplanar with the given vector so

$$\therefore \begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

So, $3x = y + z$ .....(1)

$\therefore\ \vec{r} \perp \hat{i} + \hat{j} + \hat{k}$

So, $\vec{r}\,.\,(\hat{i} + \hat{j} + \hat{k}) = 0$

So, $x + y + z = 0$ .....(2)

On solving (1) & (2)

So, $x = 0$ $\quad\therefore\ y + z = 0 \ \therefore$ (A) & (D) Satisfy

29.[D] $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$, $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

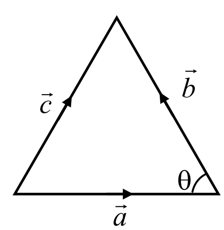
$\because\ \vec{r}\cdot\vec{a} = 0$

$\Rightarrow\ \vec{a}.\vec{c} + \lambda\vec{b}.\vec{a} = 0$

$\Rightarrow\ \lambda = -\frac{\vec{a}.\vec{c}}{\vec{b}.\vec{a}} = 4$

$\Rightarrow\ \vec{r}.\vec{b} = \vec{c}.\vec{b} + \lambda|\vec{b}|^2 = 9$

30.[A]



$$\cos\theta = \frac{-\vec{a}\cdot\vec{b}}{|-\vec{a}||\vec{b}|} = -\frac{1}{2} \ \Rightarrow\ \theta = \frac{2\pi}{3}$$

31.[3] $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

a.b + b.c + c.a = –3/2 ....(1)

$\because$ $|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0$ ....(2)

a.b + b.c + c.a $\geq \frac{-3}{2}$ ....(3)

$\because$ from (1) & (3)

so $|\vec{a}+\vec{b}+\vec{c}| = 0$

$\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{a} = -\vec{b}-\vec{c}$

on squaring

1 = 2 + 2 cos B

$\cos B = -\frac{1}{2}$ $\forall$ B = $\vec{b} \wedge \vec{c}$.

Let T = $|2\vec{a} + 5\vec{b} + 5\vec{c}|$

$= |3\vec{b} + 3\vec{c}|$

$= 3|\vec{b}+\vec{c}|$

$= 3\sqrt{2+2\cos B}$

= 3

32.[C] $|\vec{a}+\vec{b}| = \sqrt{29}$

$\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$

$(\vec{a}+\vec{b}) \times (2\hat{i}+3\hat{j}+4\hat{k}) = \vec{0}$

$\vec{a}+\vec{b} = \lambda\ (2\hat{i}+3\hat{j}+4\hat{k})$

$|\vec{a}+\vec{b}| = \sqrt{4\lambda^2+9\lambda^2+16\lambda^2} = |\lambda|\ \sqrt{29}$

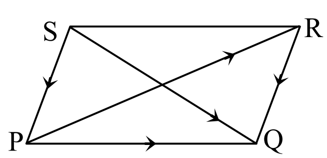
$\Rightarrow \lambda = 1, -1$

$\vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$(\vec{a}+\vec{b})\ .\ (-7\hat{i}+2\hat{j}+3\hat{k})$

$= \pm(2\hat{i}+3\hat{j}+4\hat{k})\ .(-7\hat{i}+2\hat{j}+3\hat{k}) = \pm 4$

33.[C]



$\overrightarrow{PQ}+\overrightarrow{QR} = 3\hat{i}+\hat{j}-2\hat{k}$ ….(i)

$\overrightarrow{PQ}+\overrightarrow{RQ} = \hat{i}-3\hat{j}-4\hat{k}$ ….(ii)

$\overrightarrow{PQ}-\overrightarrow{QR} = \hat{i}-3\hat{j}-4\hat{k}$ ….(iii)

$2\overrightarrow{QR} = 4\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{QR} = 2\hat{i}-\hat{j}-3\hat{k}$ ….(iv)

$\overrightarrow{PQ} = (3\hat{i}+\hat{j}-2\hat{k})-(2\hat{i}-\hat{j}-3\hat{k})$

$\overrightarrow{PQ} = \hat{i}+2\hat{j}+\hat{k}$

$\overrightarrow{PT} = \hat{i}+2\hat{j}+3\hat{k}$ (given)

and

$\overrightarrow{PQ} = \hat{i}+2\hat{j}+\hat{k}$

$\overrightarrow{QR} = 2\hat{i}-\hat{j}-3\hat{k}$

$\therefore$ Volume = $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{vmatrix}$

= 1(– 6 +1) – 2(– 3 –2) + 3(– 1 – 4)

= – 5 +10 – 15 = – 10

= 10

34.[C] (P) $[\vec{a}\,\vec{b}\,\vec{c}] = 2$

$2(\vec{a}\times\vec{b})\cdot[3(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$= 6(\vec{a}\times\vec{b})\cdot[\vec{d}\times(\vec{c}\times\vec{a})]$ (let $\vec{d} = \vec{b}\times\vec{c}$)

$= 6(\vec{a}\times\vec{b})\cdot[(\vec{d}\cdot\vec{a})\vec{c}-(\vec{d}\cdot\vec{c})\vec{a}]$

$= 6[\vec{a}\,\vec{b}\vec{c}](\vec{d}\cdot\vec{a}) - 6[\vec{a}\vec{b}\vec{a}](\vec{d}\cdot\vec{c})$

$= 6[\vec{a}\,\vec{b}\vec{c}]^2 = 6\times4 = 24$

(Q) $[\vec{a}\,\vec{b}\vec{c}] = 5$

$3(\vec{a}+\vec{b})\cdot[(\vec{b}+\vec{c})\times2(\vec{c}+\vec{a})]$

$6(\vec{a}+\vec{b})\cdot[(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})+(\vec{c}\times\vec{c})+(\vec{c}\times\vec{a})]$

$= 6([\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{c}]) = 12[\vec{a}\vec{b}\vec{c}] = 12\times5 = 60$

(R) $\frac{1}{2}|\vec{a}\times\vec{b}| = 20$

then $\frac{1}{2}|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})|$

$= \frac{1}{2}|-2(\vec{a}\times\vec{b})+3(\vec{b}\times\vec{a})|$

$= \frac{1}{2}|5(\vec{b}\times\vec{a})| = 5\times20 = 100$

(S) $|\vec{a}\times\vec{b}| = 30$

Then $|(\vec{a}+\vec{b})\times\vec{a}|$

$= |\vec{a}\times\vec{a}+\vec{b}\times\vec{a}|$

=30

35.[5] Total no. of vectors = $^8C_3$ = 56

Let consider following pairs of vectors

(i) $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}-\hat{j}-\hat{k}$

(ii) $-\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}-\hat{k}$

(iii) $\hat{i}+\hat{j}-\hat{k}$ and $-\hat{i}-\hat{j}+\hat{k}$

(iv) $\hat{i}-\hat{j}+\hat{k}$ and $-\hat{i}+\hat{j}-\hat{k}$

If we select any one pair out of these pairs and one vector from remaining 6 vectors then these 3 vectors will be coplanar.

So, total no. of coplanar vectors = $^4C_1 \times {}^6C_1$ = 24

So, total no. of non coplanar vectors = 56 – 24

= 32 = 2^5

$\therefore$ p = 5