SOLVED EXAMPLE

Ex.1 $\int \sin^2(x/2)\,dx$ equals-

(A) $\frac{1}{2}$ (x + sin x) + c (B) $\frac{1}{2}$ (x + cos x) + c

(C) $\frac{1}{2}$ (x – sin x) + c (D) None of these

Sol. Here I = $\int \frac{1-\cos x}{2}\,dx$

= $\frac{1}{2}$ (x – sin x) + c **Ans.[C]**

Ex.2 $\int \cot^2 x\,dx$ equals -

(A) – sec x + x + c (B) – cot x – x + c

(C) – sin x + x + c (D) None of these

Sol. $\int (\text{cosec}^2 x - 1)\,dx$

= – cot x – x + c **Ans. [B]**

Ex.3 $\int \frac{5x+7}{x}\,dx$ equals-

(A) 5x + 7 log x (B) 7x + 5 log x +c

(C) 5x + 7 log x + c (D) None of these

Sol. $\int \frac{5x+7}{x}\,dx = \int\left(\frac{5x}{x}+\frac{7}{x}\right)dx$

$= \int 5\,dx + \int \frac{7}{x} = 5\int 1\,dx + 7\int \frac{1}{x}\,dx$

= 5x + 7 log x + c **Ans.[C]**

Ex.4 $\int\left(x-\frac{1}{x}\right)^3 dx$, (x > 0) equals-

(A) $\frac{x^3}{3} - \frac{3}{2}x^2 + 3\log x + \frac{1}{2x^2} + c$

(B) $\frac{x^4}{3} - \frac{3}{2}x^2 + 3\log x + \frac{1}{2x^2} + c$

(C) $\frac{x^4}{4} + 3\log x + \frac{1}{2x^2} + c$

(D) None of these

Sol. $\int\left(x-\frac{1}{x}\right)^3 dx$

$= \int\left(x^3 - 3x^2.\frac{1}{x} + 3x.\frac{1}{x^2} - \frac{1}{x^3}\right)Edx$

[$\because (a-b)^3 = (a^3 - 3a^2b + 3ab^2 - b^3)$]

$= \int\left(x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}\right)dx$

$= \int x^3\,dx - 3\int x\,dx + 3\int \frac{1}{x}\,dx - \int \frac{1}{x^3}\,dx$

$= \frac{x^{3+1}}{3+1} - 3.\frac{x^{1+1}}{1+1} + 3\log x - \frac{x^{-3+1}}{-3+1} + c$

$= \frac{x^4}{4} - \frac{3}{2}x^2 + 3\log x + \frac{1}{2x^2} + c$ **Ans.[B]**

Ex.5 The value of $\int\left(\frac{6}{1+x^2} + 10^x\right)dx$ is -

(A) $6\tan^{-1}x + 10^x \log_e 10 + c$

(B) $6\tan^{-1}x + \frac{10^x}{\log_e 10} + c$

(C) $3\tan^{-1}x + \frac{10^x}{\log_e 10} + c$

(D) None of these

Sol. $\int\left(\frac{6}{1+x^2} + 10^x\right)dx$

$= 6\int \frac{1}{1+x^2}\,dx + \int 10^x dx$

$= 6\tan^{-1}x + \frac{10^x}{\log_e 10} + C$ **Ans.[B]**

Ex.6 $\int (\tan x + \cot x)^2\,dx$ is equal to-

(A) tan x – cot x + c (B) tanx + cotx + c

(C) cot x – tan x + c (D) None of these

Sol. $I = \int (\tan^2 x + \cot^2 x + 2)\,dx$

$= \int (\sec^2 x + \text{cosec}^2 x)\,dx$

= tan x – cot x + c **Ans. [A]**

Ex.7 $\int \sin 2x \sin 3x\,dx$ equals-

(A) $\frac{1}{2}$ (sin x – sin 5 x) + c

(B) $\frac{1}{10}$ (sin x – sin 5x) + c

(C) $\frac{1}{10}$ (5 sin x – sin 5x) + c

(D) None of these

Sol. $I = \frac{1}{2}\int[\cos(-x)-\cos 5x]dx$

$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right] + c$

$= \frac{1}{10}$ [5 sin x – sin 5x] + c **Ans. [C]**

Ex.8 $\int \frac{x^2}{x^2-1}$ dx equals-

(A) $x + \log\sqrt{\frac{x-1}{x+1}} + c$ (B) $x + \log\sqrt{\frac{x+1}{x-1}} + c$

(C) $x + \log\left(\frac{x-1}{x+1}\right) + c$ (D) $x + \log\left(\frac{x+1}{x-1}\right) + c$

Sol. $\int \frac{x^2-1+1}{x^2-1}dx$

$= \int\left(1+\frac{1}{x^2-1}\right)dx$

$= x + \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + c$

$= x + \log\sqrt{\frac{x-1}{x+1}} + c$ **Ans.[A]**

Ex.9 $\int \sec^2 (ax + b)$ dx equals-

(A) tan (ax + b) + c (B) $\frac{1}{2}$ tan x + c

(C) $\frac{1}{a}$ tan (ax + b) + c (D) None of these

Sol. $\int \sec^2 (ax + b)$ dx, putting ax + b = t,

adx + 0 = dt or $dx = \frac{dt}{a}$

$\therefore \int \sec^2 (ax + b)\, dx = \int \sec^2 t \frac{dt}{a}$

$= \frac{1}{a}\int \sec^2 t\, dt$

$= \frac{1}{a}$ tan t + c

$= \frac{1}{a}$ tan (ax + b) + c

(Putting the value of t) **Ans.[C]**

Ex.10 $\int \frac{1}{x \log x}$ dx is equal to-

(A) log (x log x) + c (B) log (log x + x) + c

(C) log x + c (D) log (log x) + c

Sol. $\int \frac{1}{x\log x}dx = \int \frac{1}{x} \cdot \frac{1}{\log x}dx$

put log x = t, $\frac{1}{x}dx = dt$

$\therefore \int \frac{1}{x} \cdot \frac{1}{\log x}dx = \int \frac{1}{t}\, dt$

$\therefore \int \frac{1}{t}dt$ = log t + c = log (log x) + c

(putting the value of t = log x) **Ans.[D]**

Ex.11 $\int \sec^2 x \cos(\tan x)$ dx equals-

(A) sin (cos x) + c (B) sin (tan x) + c

(C) cosec (tan x) + c (D) None of these

Sol. Let tan x = t, then $\sec^2 x\, dx = dt$

$\therefore I = \int \cos t\, dt = \sin t + c$

= sin (tan x) + c **Ans.[B]**

Ex.12 $\int \tan^n x \sec^2 x$ dx equals-

(A) $\frac{\tan^{n-1} x}{n-1} + c$ (B) $\frac{\tan^{n-1} x}{n+1} + c$

(C) $\tan^{n+1} x + c$ (D) None of these

Sol. $\int \tan^n x \sec^2 x\, dx$

putting tan x = t, $\sec^2 x\, dx = dt$

$\int \tan^n x \sec^2 x\, dx = \int t^n\, dt = \frac{\tan^{n+1}}{n+1} + c$

$= \frac{(\tan x)^{n+1}}{n+1} + c$ **Ans.[B]**

Ex.13 $\int \frac{\sin 2x}{1+\cos^4 x}$ dx is equal to-

(A) $\cos^{-1}(\cos^2 x) + c$ (B) $\sin^{-1}(\cos^2 x) + c$

(C) $\cot^{-1}(\cos^2 x) + c$ (D) None of these

Sol. Here differential coefficient of

$\cos^2 x$ is $-\sin 2x$

Let $\cos^2 x = t$

$\therefore\ 2\cos x(-\sin x)\,dx = dt$

or $\sin 2x\,dx = -dt$

$$\therefore \int \frac{\sin 2x}{1+\cos^4 x}dx = \int \frac{-dt}{1+t^2}$$

$= \cot^{-1} t + c$

$= \cot^{-1}(\cos^2 x) + c$ **Ans.[C]**

Ex.14 $\int \frac{be^x}{\sqrt{a+be^x}}dx$ equals-

(A) $\frac{2}{b}\sqrt{a+be^x} + c$ (B) $\frac{1}{b}.\sqrt{a+be^x} + c$

(C) $2\sqrt{a+be^x} + c$ (D) None of these

Sol. $\int \frac{be^x}{\sqrt{a+be^x}}dx$, putting $a + be^x = t$

$be^x\,dx = dt$

$$\therefore \int \frac{be^x}{\sqrt{a+be^x}}dx = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + c$$

$= 2\sqrt{a+be^x} + c$ **Ans.[C]**

Ex.15 $\int \sqrt{\frac{1+\cos x}{1-\cos x}}\,dx$ equals-

(A) $\log\cos\left(\frac{x}{2}\right) + c$ (B) $2\log\sin\left(\frac{x}{2}\right) + c$

(C) $2\log\sec\left(\frac{x}{2}\right) + c$ (D) None of these

Sol. $I = \int \sqrt{\frac{1+\cos x}{1-\cos x}}\,dx$

$$= \int \sqrt{\frac{2\cos^2(x/2)}{2\sin^2(x/2)}}\,dx$$

$$= \int \cot\left(\frac{x}{2}\right)dx$$

$= 2\log\sin\left(\frac{x}{2}\right) + c$ **Ans.[B]**

Ex.16 $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ equals-

(A) $2\sqrt{\sec x} + c$ (B) $2\sqrt{\tan x} + c$

(C) $2/\sqrt{\tan x} + c$ (D) $2/\sqrt{\sec x} + c$

Sol. $I = \int \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\,dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}}dx = 2\sqrt{\tan x} + c$ **Ans. [B]**

Ex.17 $\int \sin^5 x.\cos^3 x\,dx$ is equal to-

(A) $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$ (B) $\frac{\cos^6 x}{6} - \frac{\cos^8 x}{8} + c$

(C) $\frac{\cos^6 x}{6} - \frac{\sin^8 x}{8} + c$ (D) None of these

Sol. $\int \sin^5 x\,.\cos^3 x\,dx$

Assumed that $\sin x = t$

$\therefore \cos x\,dx = dt$

$= \int t^5(1-t^2)\,dt = \int (t^5 - t^7)\,dt$

$= \frac{t^6}{6} - \frac{t^8}{8} + c$

$= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$ **Ans.[A]**

Ex.18 $\int \frac{x^2}{1+x^6}dx$ is equal to-

(A) $\tan^{-1}x^3 + c$ (B) $\tan^{-1}x^2 + c$

(C) $\frac{1}{3}\tan^{-1}x^3 + c$ (D) $3\tan^{-1}x^3 + c$

Sol. Put $x^3 = t \Rightarrow x^2\,dx = \frac{1}{3}\,dt$

$\therefore I = \frac{1}{3}\int \frac{dt}{1+t^2} = \frac{1}{3}\tan^{-1}x^3 + c$ **Ans. [C]**

Ex.19 $\int \sqrt{\frac{1+x}{1-x}}\,dx$ equals-

(A) $\sin^{-1}x + \sqrt{1-x^2} + c$

(B) $\sin^{-1}x + \sqrt{x^2-1} + c$

(C) $\sin^{-1}x - \sqrt{1-x^2} + c$

(D) $\sin^{-1}x - \sqrt{x^2-1} + c$

Sol $I = \int \sqrt{\frac{1+x}{1-x}}\,dx$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int \frac{-2x\,dx}{\sqrt{1-x^2}}$$

$= \sin^{-1} x - \sqrt{1-x^2} + c$ **Ans. [C]**

Ex.20 The primitive of log x will be-

(A) x log (e + x) + c (B) $x \log\left(\frac{e}{x}\right) + c$

(C) $x \log\left(\frac{x}{e}\right) + c$ (D) x log (ex) + c

Sol. $\int \log x \, dx = \int \log x . 1 dx$

[Integrating by parts, taking log x as first part and 1 as second part]

$= (\log x).x - \int \left\{\frac{d(\log x)}{dx}\right\} . x\, dx$

$= x \log x - \int \frac{1}{x} .x\, dx = (x \log x - x) + c$

$= x (\log x - 1) + c = \log\left(\frac{x}{e}\right) + c$ **Ans. [C]**

Ex.21 $\int x \tan^{-1} x$ is equal to-

(A) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - x + c$

(B) $\frac{1}{2}(x^2 + 1) \tan^{-1} x + x + c$

(C) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2} x + c$

(D) $\frac{1}{2}(x^2 - 1) \tan^{-1} x - \frac{1}{2} x + c$

Sol. Integrating by parts taking x as second part

$I = \frac{x^2}{2} \tan^{-1} x - \int \frac{1}{1+x^2} . \frac{x^2}{2} dx$

$= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2}\left(1 - \frac{1}{1-x^2}\right) dx$

$= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + c$

$= \frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2} x + c$ **Ans. [C]**

Ex.22 $\int \sin (\log x)\, dx$ equals-

(A) $\frac{x}{\sqrt{2}} \sin (\log x + \frac{\pi}{8}) + c$

(B) $\frac{x}{\sqrt{2}} \sin (\log x - \frac{\pi}{4}) + c$

(C) $\frac{x}{\sqrt{2}} \cos (\log x - \frac{\pi}{4}) + c$

(D) None of these

Sol. $\int \sin (\log x)\, dx$, assumed that $x = e^t$

$\therefore dx = e^t dt$

$= \int \sin t.e^t.dt$

$= \frac{e^t}{\sqrt{1+1}} \sin(t - \tan^{-1} 1) + c$

$\Rightarrow \int \sin (\log x)\, dx$

$= \frac{x}{\sqrt{2}} \sin (\log x - \frac{\pi}{4}) + c$ **Ans. [B]**

Ex.23 $\int \frac{xe^x}{(x+1)^2} dx$ is equal to-

(A) $\frac{e^x}{(x+1)^2} + c$ (B) $\frac{e^x}{x+1} + c$

(C) $\frac{e^x}{(x+1)^2} + c$ (D) None of these

Sol. $I = \int e^x \left[\frac{(x+1)-1}{(x+1)^2}\right] dx$

$= \int e^x \left(\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right) dx$

$= e^x f(x) + c$

$= \frac{e^x}{x+1} + c$ **Ans. [B]**

Ex. 24 $\int x^3 (\log x)^2\, dx$ equals-

(A) $\frac{1}{32} x^4 [8 (\log x)^2 - 4 \log x + 1] + c$

(B) $\frac{1}{32} x^4 [8 (\log x)^2 - 4 \log x - 1] + c$

(C) $\frac{1}{32} x^4 [8 (\log x)^2 + 4 \log x + 1] + c$

(D) None of these

Sol. Integrating by parts taking x^3 as second part

$I = \frac{1}{4} x^4 (\log x)^2 - \frac{1}{2} \int x^3 \log x\, dx$

$= \frac{1}{4} x^4 (\log x)^2 - \frac{1}{2}\left(\frac{1}{4} x^4 \log x - \frac{1}{16} x^4\right) + c$

$= \frac{1}{32} x^4 [8 (\log x)^2 - 4 \log x + 1] + c$

Ans. [A]

Ex.25 The value of $\int x \sec x \tan x \, dx$ is-

(A) x sec x + log (sec x + tan x) + c

(B) x sec x – log (sec x – tan x) + c

(C) x sec x + log (sec x – tan x) + c

(D) None of the above

Sol. $\int x.(\sec x \tan x)\, dx$

$= (x.\sec x) - \int (1.\sec x)\, dx$

(Integrating by parts, taking x as first function)

$= x \sec x - \log (\sec x + \tan x) + c$

$= x \sec x - \log\left\{(\sec x + \tan x)\dfrac{\sec x - \tan x}{\sec x - \tan x}\right\} + c$

$= x \sec x - \log\left(\dfrac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right) + c$

$= x \sec x + \log (\sec x - \tan x) + c$ **Ans. [C]**

Ex.26 $\int \dfrac{x + \sin x}{1 + \cos x} dx$ equals-

(A) $\dfrac{x}{2}\tan\left(\dfrac{x}{2}\right) + c$ (B) $\dfrac{x}{2}\tan x + c$

(C) $x \tan\left(\dfrac{x}{2}\right) + c$ (D) $x \tan x + c$

Sol. $I = \int \dfrac{x + 2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)} dx$

$= \dfrac{1}{2}\int x \sec^2(x/2)\, dx + \int \tan(x/2)\, dx$

$= x \tan(x/2) - \int \tan(x/2)\, dx + \int \tan(x/2)\, dx$

$= x \tan\left(\dfrac{x}{2}\right) + c.$ **Ans.[C]**

Ex.27 $\int e^x \dfrac{x-1}{(x+1)^3} dx$ equals-

(A) $-\dfrac{e^x}{x+1} + c$ (B) $\dfrac{e^x}{x+1} + c$

(C) $\dfrac{e^x}{(x+1)^2} + c$ (D) $-\dfrac{e^x}{(x+1)^2} + c$

Sol. $I = \int e^x \left[\dfrac{x+1-2}{(x+1)^3}\right] dx$

$= \int e^x \left(\dfrac{1}{(x+1)^2} - \dfrac{2}{(x+1)^3}\right) dx$

Thus the given integral is of the form

$= \int e^x \{f(x) + f'(x)\}\, dx$

$\therefore\ I = e^x f(x) = \dfrac{e^x}{(x+1)^2} + c$ **Ans.[C]**

Ex. 28 $\int \sec^3\theta d\theta$ is equal to-

(A) $\dfrac{1}{2}$ [tanθ secθ + log (tanθ + secθ)] + c

(B) $\dfrac{1}{2}$ tan θ sec θ + log (tanθ + secθ) + c

(C) $\dfrac{1}{2}$ [tan θ sec θ – log (tanθ + secθ)] + c

(D) None of these

Sol. $I = \int \sec.\ \sec^2\theta.\ d\theta$

$= \int \sqrt{\tan^2\theta + 1}\sec^2\theta\, d\theta$

$= \int \sqrt{t^2 + 1}\, dt$, where $t = \tan\theta$

$= \dfrac{t}{2}\sqrt{t^2+1} + \dfrac{1}{2}\log\left(t + \sqrt{t^2+1}\right) + c$

$= \dfrac{1}{2}$ [tanθ secθ + log (tanθ + secθ)] + c

Ans. [A]

Ex.29 $\int \dfrac{\cos x + x \sin x}{x(x + \cos x)} dx$ is equal to-

(A) log {x (x + cos x)}+ c

(B) $\log\left(\dfrac{x}{x + \cos x}\right) + c$

(C) $\log\left(\dfrac{x + \cos x}{x + \cos x}\right) + c$

(D) None of these

Sol. $I = \int \dfrac{(x + \cos x) - x + x \sin x}{x(x + \cos x)} dx$

$= \int \dfrac{1}{x} dx - \int \dfrac{1 - \sin x}{x + \cos x} dx$

$= \log x - \log (x + \cos x) + c$

$= \log\left(\dfrac{x}{x + \cos x}\right) + c$ **Ans. [B]**

Ex.30 $\int \sqrt{\sec x - 1}\, dx$ is equal to-

(A) $2 \sin^{-1}(\sqrt{2}\cos x/2) + c$

(B) $-2 \sinh^{-1}(\sqrt{2}\cos x/2) + c$

(C) $-2 \cosh^{-1}(\sqrt{2}\cos x/2) + c$

(D) None of these

Sol. $I = \int \sqrt{\frac{1-\cos x}{\cos x}}\, dx$

$= \int \frac{\sqrt{2}\sin x/2}{\sqrt{2\cos^2 x/2 - 1}}\, dx$

$= -2 \int \frac{dt}{\sqrt{t^2 - 1}}$ where $t = \sqrt{2}\cos x/2$

$= -2 \cosh^{-1} t + c$

$= -2 \cosh^{-1} + (\sqrt{2}\cos x/2) + c$ **Ans. [C]**

Ex.31 $\int \frac{x^2+1}{(x-1)(x-2)}\, dx$ equals-

(A) $\log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$

(B) $x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$

(C) $x + \log\left[\frac{(x-1)^5}{(x-2)^5}\right] + c$

(D) None of these

Sol. Here since the highest powers of x in Num[r] and Den[r] are equal and coefficients of x^2 are also equal, therefore

$\frac{x^2+1}{(x-1)(x-2)} \equiv 1 + \frac{A}{x-1} + \frac{B}{x-2}$

On solving we get A = – 2, B = 5

Thus $\frac{x^2+1}{(x-1)(x-2)} \equiv 1 - \frac{2}{x-1} + \frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non repeated linear factor in the Den[r].

Now $I = \left(1 - \frac{2}{x-1} + \frac{5}{x-2}\right) dx$

$= x - 2\log(x-1) + 5\log(x-2) + c$

$= x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$ **Ans.[B]**

Ex.32 The value of $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ is-

(A) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} - a\tan^{-1}\frac{x}{a}\right] + c$

(B) $\frac{1}{b^2-a^2}\left[a\tan^{-1}\frac{x}{b} - b\tan^{-1}\frac{x}{a}\right] + c$

(C) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} + a\tan^{-1}\frac{x}{a}\right] + c$

(D) None of these

Sol. Putting $x^2 = y$ in integrand, we obtain

$\frac{y}{(y+a^2)(y+b^2)} = \frac{1}{b^2-a^2}\left[\frac{b^2}{y+b^2} - \frac{a^2}{y+a^2}\right]$

$\therefore I = \frac{1}{b^2-a^2} . \left[\int \frac{b^2}{x^2+b^2}dx - \int \frac{a^2}{x^2+a^2}dx\right]$

$= \frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} - a\tan^{-1}\frac{x}{a}\right] + c$

Ans.[A]

Ex.33 $\int \frac{dx}{3x^2+2x+1}$ equals-

(A) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$

(B) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$

(C) $\frac{1}{\sqrt{2}}\cot^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$

(D) None of these

Sol. $I = \frac{1}{3}\frac{dx}{x^2 + \frac{2}{3}x + \frac{1}{3}}$

$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2 + \frac{2}{9}}$

$= \frac{1}{3} \times \frac{3}{\sqrt{2}} \tan^{-1} + \left(\frac{x + \left(\frac{1}{3}\right)}{\sqrt{2}/3}\right) c$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$ **Ans.[A]**

Ex.34 $\int \sqrt{1+x-2x^2}\, dx$ equals-

(A) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + c$

(B) $\frac{1}{8}(4x+1)\sqrt{1+x-2x^2} - \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + c$

(C) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32}\cos^{-1}\left(\frac{4x-1}{3}\right) + c$

(D) None of these

Sol. $I = \sqrt{2}\int \sqrt{\frac{1}{2} - \left(x^2 - \frac{x}{2}\right)}\, dx$

$= \sqrt{2}\int \sqrt{\left\{\frac{9}{16} - \left(x - \frac{1}{4}\right)^2\right\}}\, dx$

$= \sqrt{2}\left[\frac{1}{2}\left(x - \frac{1}{4}\right)\sqrt{\left\{\frac{9}{16} - \left(x - \frac{1}{4}\right)^2\right\}}\right.$

$\left. + \frac{9}{32}\sin^{-1}\left\{\frac{4}{3}\left(x - \frac{1}{4}\right)\right\}\right] + c$

$= \frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + c$

Ans. [A]

Ex.35 $\int \frac{dx}{\sqrt{3-5x-x^2}}$ equals-

(A) $\sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$

(B) $\cos^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$

(C) $\sin^{-1}(2x+5) + c$

(D) None of these

Sol. $I = \int \frac{dx}{\sqrt{\frac{37}{4} - \left(x + \frac{5}{2}\right)^2}}$

$= \sin^{-1}\left(\frac{x + 5/2}{\sqrt{37}/2}\right) + c$

$= \sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$ **Ans. [A]**

Ex.36 $\int \sqrt{e^{2x}-1}\, dx$ is equal to-

(A) $\sqrt{e^{2x}-1} + \sec^{-1}e^{2x} + c$

(B) $\sqrt{e^{2x}-1} - \sec^{-1}e^{2x} + c$

(C) $\sqrt{e^{2x}-1} - \sec^{-1}e^{x} + c$

(D) None of these

Sol. $\int \frac{e^{2x}-1}{\sqrt{e^{2x}-1}}\, dx$

$= \frac{1}{2}\int \frac{2e^{2x}}{\sqrt{e^{2x}-1}}\, dx - \int \frac{e^x}{e^x\sqrt{e^{2x}-1}}\, dx$

$= \sqrt{e^{2x}-1} - \sec^{-1} e^x + c$ **Ans.[C]**

Ex.37 $\int \sqrt{\frac{e^x+a}{e^x-a}}\, dx$ is equal to-

(A) $\cosh^{-1}\left(\frac{e^x}{a}\right) + \sec^{-1}\left(\frac{e^x}{a}\right) + c$

(B) $\sinh^{-1}\left(\frac{e^x}{a}\right) + \sec^{-1}\left(\frac{e^x}{a}\right) + c$

(C) $\tanh^{-1}\left(\frac{e^x}{a}\right) + \cos^{-1}\left(\frac{e^x}{a}\right) + c$

(D) None of these

Sol. $\int \frac{e^x + a}{\sqrt{e^{2x}-a^2}}\, dx$

$= \int \frac{e^x}{\sqrt{e^{2x}-a^2}}\, dx + a\int \frac{e^x}{e^x\sqrt{e^{2x}-a^2}}\, dx$

$= \cosh^{-1}\left(\frac{e^x}{a}\right) + \sec^{-1}\left(\frac{e^x}{a}\right) + c$ **Ans.[A]**

Ex.38 $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$ is equal to-

(A) $\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(B) $\frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(C) $4\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(D) None of these

Sol. After dividing by $\cos^2 x$ to numerator and denominator of integration

$$I = \int \frac{\sec^2 x\,dx}{4\tan^2 x + 4\tan x + 5}$$

$$= \int \frac{\sec^2 x\,dx}{(2\tan x + 1)^2 + 4}$$

$$= \frac{1}{2.2}\tan^{-1}\left(\frac{2\tan x + 1}{2}\right) + c$$ **Ans. [B]**

Ex.39 $\int \left(\frac{1-x}{1+x}\right)^2 dx$ is equal to-

(A) $x - 4\log(x+1) + \frac{4}{x+1} + c$

(B) $x - \log(x+1) + \frac{4}{x+1} + c$

(C) $x - 4\log(x+1) - \frac{4}{x+1} + c$

(D) $x + \log(x+1) - \frac{4}{x+1} + c$

Sol. $\int \frac{[2-(x+1)]^2}{(x+1)^2} dx$

$$= \int \left[\frac{4}{(x+1)^2} - \frac{4}{x+1} + 1\right] dx$$

$$= -\frac{4}{x+1} - 4\log(x+1) + x + c$$ **Ans. [C]**

Ex.40 $\int \frac{e^x}{e^{2x} + 5e^x + 6}$ equals-

(A) $\log\left(\frac{e^x+3}{e^x+2}\right) + c$

(B) $\log\left(\frac{e^x+2}{e^x+3}\right) + c$

(C) $\frac{1}{2}\log\left(\frac{e^x+2}{e^x+3}\right) + c$

(D) None of these

Sol. Put $e^x = t \Rightarrow e^x dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 5t + 6} = \int \frac{dt}{(1+2)(t+3)}$$

$$= \int \left(\frac{1}{t+2} - \frac{1}{t+3}\right) dt$$

$$= \log\left(\frac{t+2}{t+3}\right) + c = \log\left(\frac{e^x+2}{e^x+3}\right) + c$$

Ans. [B]

Ex.41 $\int \frac{dx}{x + \sqrt{x}}$ equals-

(A) $2\log(\sqrt{x} - 1) + c$ (B) $2\log(\sqrt{x} + 1) + c$

(C) $\tan^{-1} x + c$ (D) None of these

Sol. $I = \int \frac{dx}{x + \sqrt{x}}$

$$= \int \frac{2t\,dt}{t^2 + t} \text{ where } t^2 = x$$

$$= 2\int \frac{dt}{t+1} = 2\log(\sqrt{x} + 1) + c$$ **Ans.[B]**

Ex.42 $I = \int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$ dx is equal to-

(A) $\frac{19}{36}x + \frac{35}{36}\log(9e^x - 4e^{-x}) + c$

(B) $-\frac{19}{36}x + \frac{35}{36}\log(9e^x - 4e^{-x}) + c$

(C) $\frac{1}{36}x + \frac{1}{36}\log(9e^x - 4e^{-x}) + c$

(D) None of these

Sol. Suppose $4e^x + 6e^{-x} = A(9e^x - 4e^{-x}) + B(9e^x + 4e^{-x})$

By comparing $4 = 9A + 9B$,

$6 = -4A + 4B$

or $A + B = \frac{4}{9}$, $-A + B = \frac{3}{2}$

After solving $A = -\frac{19}{36}$, $B = \frac{35}{36}$

$$\therefore I = \int \left[-\frac{19}{36} + \frac{35}{36}\left(\frac{9e^x + 4e^{-x}}{9e^x - 4e^{-x}}\right)\right] dx$$

$$= -\frac{19}{36}x + \frac{35}{36}\log(9e^x - 4e^{-x}) + c$$

Ans.[B]

Ex.43 $\int \frac{\sin^{-1}\sqrt{x}}{\sqrt{1-x}}$ dx equals-

(A) $2[\sqrt{x} - \sqrt{1-x}\sin^{-1}\sqrt{x}] + c$

(B) $2[\sqrt{x} + \sqrt{1-x}\sin^{-1}\sqrt{x}] + c$

(C) $[\sqrt{x} - \sqrt{1-x}\sin^{-1}\sqrt{x}] + c$

(D) None of these

Sol. Let $x = \sin^2 t$, then

$dx = 2\sin t\cos t\,dt$

$$\therefore I = \int \frac{t}{\cos t} . 2\sin t\cos t\,dt$$

$$= 2\int t\sin t\,dt$$

$$= 2[-t\cos t + \sin t] + c$$

$= 2[\sqrt{x} - \sqrt{1-x}\,\sin^{-1}\sqrt{x}\,] + c$

Ans. [A]

Ex.44 $\int \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x+a}}\,dx$ equals-

(A) $\sqrt{x^2+ax} - 2\sqrt{ax+a^2} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

(B) $\sqrt{x^2+ax} + \sqrt{ax+a^2} - a\cosh^{-1} + \left(\sqrt{\frac{x+a}{a}}\right) c$

(C) $\sqrt{x^2+ax} - 2\sqrt{ax+a^2} + a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

(D) None of these

Sol. Let $x = a\tan^2\theta \Rightarrow dx = 2a\tan\theta\sec^2\theta\,d\theta$

$\therefore I = \int \frac{\sqrt{a}(\tan\theta - 1).2a\tan\theta\sec^2\theta}{\sqrt{a}\sec\theta}\,d\theta$

$= 2a\,[\int \tan^2\theta\sec\theta\,d\theta - \int \sec\theta\tan\theta\,d\theta\,]$

$= 2a\,[\int \sqrt{\sec^2\theta - 1}\,\tan\theta\sec\theta\,d\theta - \sec\theta]$

$= 2a\int\sqrt{t^2-1}\,dt - 2a\sec\theta + c$ [Where $\sec\theta = t$]

$= 2a\left[\frac{t}{2}\sqrt{t^2-1} - \frac{1}{2}\cosh^{-1}(t)\right] - 2a\sqrt{\frac{a+x}{a}} + c$

$= a\sqrt{\frac{x+a}{a}.\frac{x}{a}} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right)$

$- 2\sqrt{ax+a^2} + c$

$= \sqrt{x^2+ax} - 2\sqrt{ax+a^2} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

Ans. [A]

Ex.45 $\int \frac{x^5}{\sqrt{1+x^3}}\,dx$ equals-

(A) $\frac{2}{9}(x^3-2)\sqrt{1+x^3} + c$

(B) $\frac{2}{9}(x^3+2)\sqrt{1+x^3} + c$

(C) $(x^3+2)\sqrt{1+x^3} + c$

(D) None of these

Sol. Put $1 + x^3 = t^2 \Rightarrow 3x^2\,dx = 2t\,dt$

$\therefore I = \int \frac{x^3}{\sqrt{1+x^3}}(x^2\,dx) = \frac{2}{3}\int (t^2-1)\,dt$

$= \frac{2}{3}\left[\frac{t^3}{3} - t\right] + c$

$= \frac{2}{3}\left[\frac{1}{3}(1+x^3)^{3/2} - \sqrt{1+x^3}\right] + c$

$= \frac{2}{9}\sqrt{1+x^3}\,(1+x^3-3) + c$

$= \frac{2}{9}(x^3-2)\sqrt{1+x^3} + c$ **Ans. [A]**

Ex. 46 $\int \frac{e^{2\tan^{-1}x}(1+x)^2}{(1+x^2)}\,dx$ is equal to-

(A) $xe^{\tan^{-1}x} + c$ (B) $xe^{2\tan^{-1}x} + c$

(C) $2xe^{2\tan^{-1}x} + c$ (D) None of these

Sol. Putting the value of $2\tan^{-1}x = t$

$I = \frac{1}{2}\int e^t\{1+\tan(t/2)\}^2\,dt$

$= \frac{1}{2}\int e^t\left[\sec^2\frac{t}{2} + 2\tan\frac{t}{2}\right]dt$

$= \frac{1}{2}\,e^t(2\tan t/2)$

$= e^t\tan\frac{t}{2} = xe^{2\tan^{-1}x} + c$ **Ans. [B]**

Ex.47 If $I = \int \cos^{-1}\sqrt{x}\,dx$ and

$J = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$, then J equals-

(A) $x - 4I$ (B) $x + I$

(C) $x - \frac{4}{\pi}I$ (D) $\frac{\pi}{4}$

Sol. Here

$J = \frac{2}{\pi}\int\{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\}\,dx$

$= \frac{2}{\pi}\left(\frac{\pi}{2} - 2\cos^{-1}\sqrt{x}\right)dx$

$[\because \sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}\,]$

$= \int dx - \frac{4}{\pi}\int\cos^{-1}\sqrt{x}\,dx$

$= x - \frac{4}{\pi}I.$ **Ans. [C]**

Ex.48 Which value of the constant of integration will make the integral of sin 3x cos 5x zero at x = 0

(A) 0 (B) – 3/16

(C) –5/16 (D) 1/8

Sol. $I = \frac{1}{2}\int(\sin 8x - \sin 2x)\,dx$

$= \frac{1}{2}\left[-\frac{\cos 8x}{8} + \frac{\cos 2x}{2}\right] + c$

At x = 0, $I = -\frac{1}{16} + \frac{1}{4} + c$

$\therefore$ I = 0 ⇒ c = – 3/16 **Ans. [B]**

Ex.49 If $\int \frac{dx}{1+\sin x} = \tan\left(\frac{x}{2} + a\right) + c$, then value of a is

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) π (D) $\frac{\pi}{2}$

Sol. $I = \int \frac{dx}{1+\sin x}$

$= \int \frac{dx}{1+\cos\{(\pi/2)-x\}}$

$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)dx$

$= -\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) + c = \tan\left(-\frac{\pi}{4} + \frac{x}{2}\right) + c$

$\therefore \alpha = -\frac{\pi}{4}$ **Ans. [B]**

Ex. 50 If $\int \frac{2x+3}{(x-1)(x^2+1)}\,dx = \log[(x-1)^{5/2}(x^2+1)^a]$ $-\frac{1}{2}\tan^{-1}x + k$ where k is any arbitrary constant, then a is equal to

(A) 5/4 (B) –5/3

(C) –5/6 (D) –5/4

Sol. Let $\frac{2x+3}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

⇒ $2x + 3 = A(x^2+1) + (Bx + C)(x-1)$...(1)

Now putting x = 1, we get 5 = 2A ⇒ A = 5/2

Equating coefficients of similar terms on both sides of (1),

we get,

– B + C = 2, A – C = 3

⇒ C = 5/2 – 3 = – 1/2

B = – 1/2 – 2 = – 5/2

$\therefore I = \frac{5}{2}\int\frac{dx}{x-1} + \int\frac{-\frac{5}{2}x - \frac{1}{2}}{x^2+1}\,dx$

$= \frac{5}{2}\log(x-1) - \frac{5}{4}\int\frac{2x}{x^2+1}\,dx - \frac{1}{2}\int\frac{1}{x^2+1}\,dx$

$= \frac{5}{2}\log(x-1) - \frac{5}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + c$

$= \log[(x-1)^{5/2}(x^2+1)^{-5/4}] - \frac{1}{2}\tan^{-1}x + c$

$\therefore$ a = – 5/4. **Ans. [D]**