MATHEMATICS

Class-VII

Topic-03

RATIONAL NUMBER

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CHAPTER 03

RATIONAL NUMBER

TERMINOLOGIES

Rational Number, Standard Form, Positive Rational Numbers, Negative Rational Numbers, Equivalent Rational numbers, Lowest Form, Irrational Numbers, Real Numbers..

INTRODUCTION

Till now we have studied that a distance of 750 m above sea level can be represented as 3/4km , but can we also represent distance of 3/4 km below the sea level as – 3/4 km? So lets deal with this problem in this chapter.

The term " rational " arises from the term 'ratio'. As we know that ratio like 2 : 5 can also be written as $\frac{2}{5}$. So numbers that can be expressed in the form $\frac{p}{q}$, where q is a non–zero integer and p is any integer are called rational numbers.

3.1 RATIONAL NUMBER

(a) Some results on Rational Numbers

(i) Every integer is a rational number but a rational number need not be an integer.

Explanation : We know that

$1 = \frac{1}{1}$ and so on. Also. $-1 = \frac{-1}{1}$ and so on.

(ii) Every fraction is a rational number but a rational number need not be a fraction .

We know that $\frac{3}{-5}$ is a rational number but it is not a fraction because its denominator is not a natural number.

(iii) A rational number $\frac{p}{q}$ is said to be in the standard form if q is a positive integer and the integers $\frac{p}{q}$ have no common divisor other than 1.

e.g. Standard form of $\frac{3}{-6}$ is $\frac{-1}{2}$.

(b) Types of Rational Number

(i) Positive Rational Number :

A rational numbers $\frac{p}{q}$ is positive, if p and q are either both positive or both negative.

Each of the rational numbers $\frac{2}{3}, \frac{5}{9}, \frac{-7}{-12}, \frac{-3}{-11}$ is a positive rational number.

(ii) Negative Rational Number :

A rational number $\frac{p}{q}$ is negative, if p and q are of opposite signs.

$\frac{-3}{7}, \frac{5}{-9}, \frac{-15}{26}$

(c) Equivalent Rational Numbers :

To convert a rational number to an equivalent rational number, either multiply or divide both its numerator and denominator by a non–zero integer.

NOTE :

(i) Two rational numbers are equal if they have the same standard form.

(ii) If $\frac{x}{y}$ is a rational number and m is any non–zero integer, then $\frac{x}{y} = \frac{x \times m}{y \times m}$.

For example, $\frac{3}{8} = \frac{3 \times 4}{8 \times 4} = \frac{12}{32}$

(iii) If $\frac{x}{y}$ is a rational number and m is a common divisor of x and y, then

$$\frac{x}{y} = \frac{x \div m}{y \div m}$$

$$\frac{-27}{45} = \frac{(-27) \div 3}{45 \div 3} = \frac{-9}{15} = \frac{(-9) \div 3}{15 \div 3} = \frac{-3}{5}$$

(iv) If x and y are positive integers, then the rational numbers $\frac{x}{y}$ and $\frac{-x}{-y}$ are both positive and the rational numbers $\frac{-x}{y}$ and $\frac{x}{-y}$ are both negative.

(v) $\frac{a}{b} = \frac{c}{d}$ only when a × d = b × c

(vi) If there are two rational numbers with common denominator, then one with the larger numerator is larger than the other. .

(vii) Every positive rational number is greater than zero.

(viii) Every negative rational number is less than zero.

(ix) Every positive rational number is greater than every negative rational number.

(d) Lowest form of a rational number

Definition : A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1.

Every rational number can be put in the lowest form using the following steps :

Step I Obtain the rational number $\frac{p}{q}$

Step II Find the HCF of p and q say m

Step III If m = 1, then $\frac{p}{q}$ is in lowest form.

Step IV If $m \neq 1$, then $\frac{p \div m}{q \div m}$ is the lowest form of $\frac{p}{q}$.

Illustration 3.1

Find whether the following rational numbers are in the lowest form or not.

(i) $\frac{17}{79}$ **(ii)** $\frac{24}{320}$

Sol. **(i)** We observe that 17 and 79 have no common factor, i.e., their HCF is 1.

Therefore, $\frac{17}{79}$ is in the lowest form.

(ii) We have,

$24 = 2 \times 2 \times 2 \times 3$ and $320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

Thus, HCF of 24 and 320 is $2 \times 2 \times 2 = 8$.

Therefore, $\frac{24}{320}$ is not in the lowest form.

Illustration 3.2

Express each of the following rational numbers to the lowest form.

(i) $\frac{12}{16}$ **(ii)** $\frac{-60}{72}$

Sol. **(i)** We have,

$12 = 2 \times 2 \times 3$ and $16 = 2 \times 2 \times 2 \times 2$

$\therefore$ HCF of 12 and 16 is $2 \times 2 = 4$.

So, $\frac{12}{16}$ is not in lowest form.

Dividing numerator and denominator by 4, we have

$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

$\therefore$ $\frac{3}{4}$ is the lowest form of $\frac{12}{16}$.

(ii) We have

$60 = 2 \times 2 \times 3 \times 5$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

$\therefore$ HCF of 60 and 72 is $2 \times 2 \times 3 = 12$

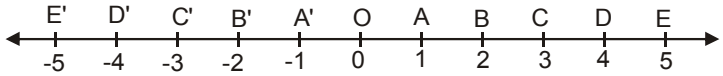
Dividing numerator and denominator of $\frac{-60}{72}$ by 12.

We get $\frac{-5}{6}$.

(e) Representation of rational number on real line

Draw any line. Take a point 0 on it. Call it 0 (zero). Set off equal distances on the right as well as on the left of 0. Such a distance is known as a unit length.

Clearly, the points A, B, C, D, E represents the integers 1, 2, 3, 4, 5 respectively and the points A', B', C', D', E', represents the integers – 1, – 2, – 3, – 4, – 5 respectively.



Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of 0 and every negative integer lies to the left of 0. Similarly we can represent rational numbers. Consider the folllowing examples :

Illustration 3.3

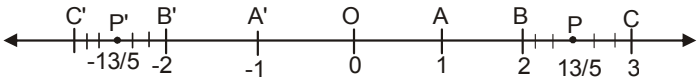
Represent $\frac{13}{5}$ and $\frac{-13}{5}$ on the number line.

Sol. Draw a line. Take a point O on it, Let it represent 0.

Now, $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$.

From 0, set off unit distances OA, AB and BC to the right of O. Clearly, the points A, B and C represent the integers 1, 2 and 3 respectively. Now, take 2 units OA and AB, and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at a point P.

Then the point P represents the rational number $\frac{13}{5}$.



Again, from O, set off unit distances to the left. Let these segments be OA', A' B', B' C', etc. Then, clearly the points A', B' and C' represent the integers –1, – 2, – 3 respectively.

Now, $\frac{-13}{5} = -\left(2 + \frac{3}{5}\right)$

Take 2 full unit lengths to the left of O. Divide the third unit B'C' into 5 equal parts.
Take 3 parts out of these 5 parts to reach a point P'.

Then, the point P' represents the rational number $\frac{-13}{5}$.

Thus, we can represent every rational number by a point on the number line.

NOTE :

(i) Every rational number represented by a point on the number line is greater than every rational number represented by points on its left.

(ii) Every rational number represented by a point on the number line is less than every rational number represented by points on its right.

(f) Comparing two Rational Numbers

In order to compare any two rational numbers, we can use the following steps :

Step I Obtain the given rational numbers.

Step II Write the given rational numbers so that their denominators are positive.

Step III Find the LCM of the positive denominators of the rational numbers obtained in step II.

Step IV Express each rational number (obtained in step II) with the LCM (obtained in step III) as common denominator.

Step V Compare the numerators of rational numbers obtained in step IV. The number having greater numerator is the greater rational number.

Illustration 3.4

Which of the two rational numbers $\frac{3}{5}$ and $\frac{-2}{3}$ is greater?

Sol. Clearly, $\frac{3}{5}$ is a positive rational number and $\frac{-2}{3}$ is a negative rational number. We know that every positive rational number is greater than every negative rational number.

$\therefore \quad \frac{3}{5} > \frac{-2}{3}$.

Illustration 3.5

Which of the two rational numbers $\frac{5}{7}$ and $\frac{3}{5}$ is greater ?

Sol. Clearly, denominators of the given rational numbers are positive. The denominators are 7 and 5. The LCM of 7 and 5 is 35. So, first express each rational number with 35 as common denominator.

$\therefore \quad \frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$ and $\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$

Now, compare the numerators of these rational numbers.

$\therefore \quad 25 > 21 \Rightarrow \frac{25}{35} > \frac{21}{35} \Rightarrow \frac{5}{7} > \frac{3}{5}$.

Illustration 3.6

Which of the two rational numbers $\frac{-4}{9}$ and $\frac{5}{-12}$ is greater ?

Sol. First write each one of the given rational numbers with positive denominator.

Clearly, denominator of $\frac{-4}{9}$ is positive. The denominator of $\frac{5}{-12}$ is negative.

So, express it with positive denominator as follows

$\frac{5}{-12} = \frac{5\times(-1)}{(-12)\times(-1)} = \frac{-5}{12}$

Write the rational numbers so that they have a common denominator 36 as follows :

$\frac{-4}{9} = \frac{-4\times4}{9\times4} = \frac{-16}{36}$ and $\frac{-5}{12} = \frac{-5\times3}{12\times3} = \frac{-15}{36}$

$\therefore \quad -15 > -16 \quad \Rightarrow \frac{-15}{36} > \frac{-16}{36} \Rightarrow \frac{-5}{12} > \frac{-4}{9} \Rightarrow \frac{5}{-12} > \frac{-4}{9}$

Illustration 3.7

Arrange the rational numbers $\frac{-7}{10}$, $\frac{5}{-8}$, $\frac{2}{-3}$ in ascending order.

Sol. First write the given rational numbers so that their denominators are positive.

$\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$ and $\frac{2}{-3} = \frac{2\times(-1)}{-3\times(-1)} = \frac{-2}{3}$

Thus, the given rational numbers with positive denominators are :

$\frac{-7}{10}, \frac{-5}{8}, \frac{-2}{3}$

Now, LCM of the denominators 10, 8 and 3 is :

$2 \times 2 \times 5 \times 2 \times 3 = 120$

Now write the numbers so that they have a common denominator 120 as follows :

$\frac{-7}{10} = \frac{-7\times12}{10\times12} = \frac{-84}{120}$, $\frac{-5}{8} = \frac{-5\times15}{8\times15} = \frac{-75}{120}$ and $\frac{-2}{3} = \frac{-2\times40}{3\times40} = \frac{-80}{120}$

Comparing the numerators of these numbers, we get

$-84 < -80 < -75$

$\therefore \quad \frac{-84}{120} < \frac{-80}{120} < \frac{-75}{120} \Rightarrow \frac{-7}{10} < \frac{-2}{3} < \frac{-5}{8}$

$\Rightarrow \quad \frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$

Ask yourself

1. Find four rational number equivalent to $\frac{-3}{5}$.

2. Express $\frac{-48}{132}$ in standard form.

3. Express $\frac{-8}{11}$ as a rational number with denominator – 55.

4. Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ in ascending order.

5. Is $\frac{7}{3}$ is greater than $\frac{9}{4}$.

Answers

1. $\frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \frac{-15}{25}$ **2.** $\frac{-4}{11}$ **3.** $\frac{40}{-55}$ **4.** $\frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$

5. Yes

3.2 OPERATION ON RATIONAL NUMBER

(a) Addition

If two rational numbers are to be added we should convert each of them into a rational number with positive denominator.

Case I : When given number have same denominator.

In this case we define $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

Illustration 3.8

Add $\frac{7}{5}$ and $\frac{9}{5}$

Sol. $\frac{7}{5} + \frac{9}{5} = \frac{7+9}{5} = \frac{16}{5}$.

Case II : When denominator of given number are unequal.

In this case we take the LCM of their denominators and express each of the given numbers with this LCM as the common denominator. Now we add these numbers as shown above.

Illustration 3.9

Add $\frac{3}{8}$ and $\frac{5}{6}$.

Sol. The denominators of the given rational numbers are 8 and 6 respectively.

LCM of 8 and 6 is 24

Now, $\frac{3}{8} = \frac{3\times3}{8\times3} = \frac{9}{24}$; $\frac{5}{6} = \frac{5\times4}{6\times4} = \frac{20}{24}$

$\therefore \quad \frac{3}{8} + \frac{5}{6} = \frac{9}{24} + \frac{20}{24} = \frac{9+20}{24} = \frac{29}{24}$.

Illustration 3.10

Find the sum : $\frac{-7}{5}+\frac{2}{3}$.

Sol. LCM of 5 and 3 = (5 × 3) = 15.

$\therefore \frac{-7}{5}+\frac{2}{3}=\frac{3\times(-7)+5\times 2}{15}=\frac{-21+10}{15}=\frac{-11}{5}$.

(i) Existence of Additive Inverse :

For every rational number $\frac{a}{b}$, there exists a rational number $\frac{-a}{b}$

such that $\left(\frac{a}{b}+\frac{-a}{b}\right)=\frac{\{a+(-a)\}}{b}=\frac{0}{b}=0$

and similarly, $\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

Thus, $\left(\frac{a}{b}+\frac{-a}{b}\right)=\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

$\frac{-a}{b}$ is called the additive inverse of $\frac{a}{b}$.

For example : $\left(\frac{4}{7}+\frac{-4}{7}\right)=\frac{\{4+(-4)\}}{7}=\frac{0}{7}=0$ and Similarly, $\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

$\therefore \quad \left(\frac{4}{7}+\frac{-4}{7}\right)=\left(\frac{4}{7}+\frac{-4}{7}\right)=0$.

Thus, $\frac{4}{7}$ and $\frac{-4}{7}$ are additive inverse of each other.

(b) Subtraction

For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$\left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\frac{-c}{d}\right)=\frac{a}{b}+\left(\text{additive inverse of }\frac{c}{d}\right)$

Illustration 3.11

Find the additive inverse of :

(i) $\frac{5}{9}$ **(ii)** $\frac{9}{-11}$

Sol. **(i)** Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(ii) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$.

Hence, its additive inverse is $\frac{9}{11}$.

Illustration 3.12

(i) $\frac{-5}{7}$ from $\frac{-2}{5}$ **(ii)** $\frac{9}{16}$ from $\frac{7}{24}$

Sol. **(i)** $\left\{\frac{-2}{5}-\left(\frac{-5}{7}\right)\right\}=\left(\frac{-2}{5}+\frac{5}{7}\right)=\frac{(-14+25)}{35}=\frac{11}{35}$.

(ii) $\frac{7}{24}-\frac{9}{16}=\frac{14-27}{48}=\frac{-13}{48}$.

Illustration 3.13

What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?

Sol. Let the required number to be added be x.

Then, $\frac{-7}{8}+x=\frac{4}{9}$

$x=\left(\frac{4}{9}+\frac{7}{8}\right)=\frac{(-32+63)}{72}=\frac{95}{72}$.

Hence, the required number is $\frac{95}{72}$.

(c) Multiplication

For any two rationals $\frac{a}{b}$ and $\frac{c}{d}$, we define : $\left(\frac{a}{b}\times\frac{c}{d}\right)=\frac{(a\times c)}{(b\times d)}$.

Illustration 3.14

Find each of the following products :

(i) $\frac{-15}{4}\times\frac{-3}{8}$ **(ii)** $\frac{3}{7}\times\frac{-5}{8}$

Sol. We have

(i) $\frac{-15}{4}\times\frac{-3}{8}=\frac{(-15)\times(-3)}{4\times8}=\frac{45}{32}$. **(ii)** $\frac{3}{7}\times\frac{-5}{8}=\frac{3x(-5)}{7x8}=\frac{-15}{56}$.

Multiplicative Inverse :

Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1$.

$\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$. Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (–1) is (–1).

For example : Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1$.

(c) Division

Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1$.

$\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$. Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (–1) is (–1).

For example : Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1$.

When $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is called dividend; $\frac{c}{d}$ is called the divisor and the result is known as quotient.

Illustration 3.15

A car is moving at an average speed of $36\frac{4}{5}$ km/h. How much distance will it cover in $7\frac{1}{2}$ hours?

Sol. Distance covered by the car in 1 hour = $36\ \frac{4}{5}$ km = $\frac{184}{5}$ km.

Distance covered by the car in $7\frac{1}{2}$ hours = $\left(\frac{184}{5}\times\frac{15}{2}\right)$ km = 276 km.

Hence, the required distance is 276 km.

Illustration 3.16

The product of two rational numbers is $\frac{-8}{9}$. If one of the numbers is $\frac{-4}{15}$. Find the other.

Sol. Let the required number be x. Then,

$$\frac{-4}{15}\times x=\frac{-8}{9} \quad\Rightarrow\quad x=\frac{-8}{9}\div\frac{-4}{15}$$

$$\Rightarrow\quad x=\frac{-8}{9}\times\frac{15}{-4}=\frac{-8}{9}\times\frac{-15}{4} \quad\Rightarrow\quad x=\frac{(-8)\times(-15)}{9\times4}$$

$$\Rightarrow\quad x=\frac{8\times15}{9\times4} \quad\Rightarrow\quad x=\frac{10}{3}.$$

Hence, the other number is $\frac{10}{3}$.

(d) Insertion of Rational number between two given Rational Numbers:

Between any two rational numbers infinitely many rational numbers can be inserted.

Illustration 3.17

Find some rational numbers lying between two given rational numbers, say between $\frac{-2}{5}$ and $\frac{3}{5}$.

Sol. The four rational numbers $\frac{-1}{5}, \frac{0}{5}, \frac{1}{5}$ and $\frac{2}{5}$ lie between $\frac{-2}{5}$ and $\frac{3}{5}$.

If required, we can insert more rational numbers between $\frac{-2}{5}$ and $\frac{3}{5}$ by the technique as discussed below.

The rational numbers $\frac{-2}{5}$ and $\frac{3}{5}$ can also be written as $\frac{-20}{50}$ and $\frac{30}{50}$ respectively

Clearly, $\frac{-19}{50}, \frac{-18}{50}, \ldots \frac{0}{50}, \ldots, \frac{29}{50}$ are rational between $\frac{-2}{5}$ and $\frac{3}{5}$.

Illustration 3.18

Insert 10 rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$.

Sol. We know that

$-3 < -2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8$

$\therefore \frac{-3}{11} < \frac{-2}{11} < \frac{-1}{11} < \frac{0}{11} < \frac{1}{11} < \frac{2}{11} < \frac{3}{11} < \frac{4}{11} < \frac{5}{11} < \frac{6}{11} < \frac{7}{11} < \frac{8}{11}$

Hence, 10 rational numbers between $\frac{-3}{13}$ and $\frac{8}{11}$:

$\frac{-2}{11}, \frac{-1}{11}, \frac{0}{11}, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}$ and $\frac{7}{11}$.

Illustration 3.19

Insert 100 rational numbers between $\frac{-3}{13}$ and $\frac{9}{13}$

Sol. We have,

$\frac{-3}{13} = \frac{-3\times10}{13\times10} = \frac{-30}{130}$ and $\frac{9}{13} = \frac{9\times10}{13\times10} = \frac{90}{130}$

We know that

$-30 < -29 < -28 < ..... -1 < 0 < 1 < 2 < 3 < .... < 90$

$\Rightarrow \frac{-30}{130} < \frac{-29}{130} < \frac{-28}{130} < ... < \frac{-1}{130} < \frac{0}{130} < \frac{1}{130} < \frac{2}{130} < .... < \frac{90}{130}$

Hence 100 rational numbers between $\frac{-3}{13} = \frac{-30}{130}$ and $\frac{9}{13} = \frac{90}{130}$ are

$\frac{-29}{130}, \frac{-28}{130}, ..., \frac{-1}{130}, \frac{0}{130}, \frac{1}{130}, ..., \frac{70}{130}$.

Ask yourself

1. What should be added to $5\frac{4}{15}$ to get $12\frac{3}{5}$?

2. Find the sum of $3\frac{4}{5} + 2\frac{7}{10} + 4\frac{8}{15}$

3. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?

4. If $3\frac{7}{9}$ m cloth cost Rs. $212\frac{4}{5}$. Find the cost of $7\frac{2}{3}$ m cloth.

5. From a rope of 40m, 50 cm, pieces of equal size are cut. If each piece is $2\frac{1}{4}$ m long. Find the number of pieces cut off ?

6. List six rational number between –2 and –1.

Answers

1. $\frac{22}{3}$ **2.** $\frac{331}{30}$ **3.** 7 **4.** $431\frac{219}{255}$ **5.** 18

6. $\frac{-13}{7}$.... $\frac{-8}{7}$

Add your knowledge

Irrational Numbers : All real number which are not rational are **irrational numbers**. These are non–recurring as well as non–terminating type of decimal numbers.

For Ex. : $\sqrt{2}$, $\sqrt[3]{4}$, $2+\sqrt{3}$, $\sqrt{2+\sqrt{3}}$, $\sqrt[4]{7\sqrt{3}}$ etc.

Irrational Number in Decimal Form :

$\sqrt{2}$ = 1.414213..... i.e. it is non–recurring as well as non–terminating.

$\sqrt{3}$ = 1.732050807..... i.e. it is non–recurring as well as non–terminating.

INSERTION OF IRRATIONAL NUMBERS BETWEEN TWO REAL NUMBERS

Let a and b are two given real numbers, then irrational number between a and b is $\sqrt{a\times b}$.

1. Insert an irrational number between 2 and 3.

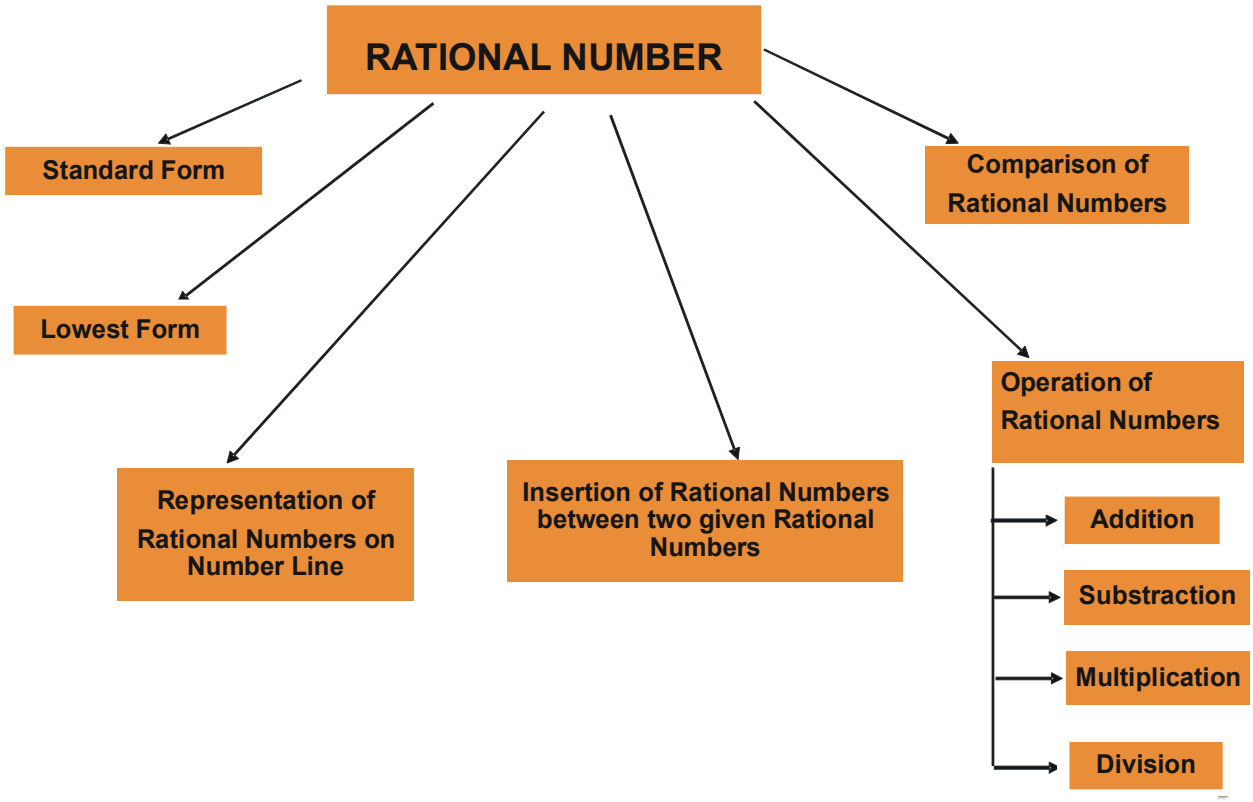
Sol. $\sqrt{2\times 3}=\sqrt{6}$

2. Find two irrational number between 2 and 2.5.

Sol. 2.101001000100001..... is between 2 and 5 and it is non–recurring as well as non–terminating.

Also, 2.201001000100001 ..... and so on.

Concept Map


RATIONAL NUMBER
Standard Form
Lowest Form
Representation of Rational Numbers on Number Line
Insertion of Rational Numbers between two given Rational Numbers
Comparison of Rational Numbers
Operation of Rational Numbers
Addition
Substraction
Multiplication
Division

Summary

1. Every integer is a rational number but a rational number need not be an integer.

2. Every fraction is a rational number but a rational number need not be a fraction

3. A rational number $\frac{p}{q}$ is said to be in the standard form if q is a positive integer and the integers $\frac{p}{q}$ have no common divisor other than 1.

4. A rational numbers is $\frac{p}{q}$ positive, if p and q are either both positive or both negative.

5. A rational number $\frac{p}{q}$ is negative, if p and q are of opposite signs.

6. If there are two rational numbers with common denominator, then one with the larger numerator is larger than the other. .

7. Every positive rational number is greater than zero.

8. Every negative rational number is less than zero.

9. Every positive rational number is greater than every negative rational number.

10. For any two rational numbers $\frac{p}{q}$ and $\frac{r}{q}$ we define :

$$\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q}$$

11. For any two rational numbers $\frac{p}{q}$ and $\frac{r}{q}$ to find $\frac{p}{q}+\frac{r}{q}$ first convert $\frac{p}{q}$ and $\frac{r}{q}$ to equivalent rational numbers having denominator equal to the LCM of q and s and then they are added.

12. For any two rational numbers $\frac{p}{q}$ and $\frac{r}{q}$, $\frac{p}{q}-\frac{r}{q}=\frac{p}{q}+\left(\text{negative of } \frac{r}{s}\right)$

13. For any two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$

$$\frac{p}{q}\times\frac{r}{s}=\frac{p\times r}{q\times s}$$

14. The reciprocal of a non–zero rational number $\frac{p}{q}$ is $\frac{q}{p}$ and written as $\left(\frac{p}{q}\right)^{-1}=\frac{q}{p}$.

15. For any two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ $(\neq 0)$,

$$\frac{p}{q}\div\frac{r}{q}=\frac{p\times s}{q\times r}$$

16. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers and $\frac{a}{b}+\frac{c}{d}=0$, then each is additive inverse (or negative) of the other

17. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers and $\frac{a}{b}\times\frac{c}{d}=1$, then each is multiplicative inverse (or reciprocal) of the other

EXERCISE 01

SECTION –A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

1. Standard form of $\frac{33}{-55}$ is :

(A) $\frac{3}{-5}$ (B) $\frac{-3}{5}$ (C) $\frac{-33}{55}$ (D) None of these

2. What will be the correct sign in the following between $\frac{1}{5}$ and $\frac{2}{3}$?

(A) = (B) > (C) < (D) $\geq$

3. Which of the following statements is true ?

(A) $-\frac{4}{9} < -\frac{7}{18} < -\frac{5}{12} < -\frac{2}{3}$ (B) $-\frac{7}{18} < -\frac{5}{12} < -\frac{4}{9} < -\frac{2}{3}$

(C) $-\frac{2}{3} < -\frac{4}{9} < -\frac{5}{12} < -\frac{7}{18}$ (D) $-\frac{5}{12} < -\frac{2}{3} < -\frac{4}{9} < -\frac{7}{18}$

4. If $\frac{x}{6} = \frac{7}{-3}$, then the value of x is :

(A) – 14 (B) 14 (C) 21 (D) –21

5. What should be added to $\frac{-5}{9}$ to get 1 ?

(A) $\frac{4}{9}$ (B) $\frac{-4}{9}$ (C) $\frac{14}{9}$ (D) $\frac{-14}{9}$

6. What should be subtracted from $\frac{-3}{4}$ to get $\frac{5}{6}$?

(A) $\frac{19}{12}$ (B) $\frac{-19}{12}$ (C) $\frac{1}{12}$ (D) $\frac{-1}{12}$

7. By what rational number should $\frac{-8}{39}$ be multiplied to obtain 26 ?

(A) $\frac{507}{4}$ (B) $\frac{-507}{4}$ (C) $\frac{-407}{4}$ (D) None

8. By what rational number should $\frac{-4}{39}$ be multiplied to obtain 18 ?

(A) $\frac{351}{2}$ (B) $\frac{-351}{2}$ (C) $\frac{-24}{13}$ (D) None of these

9. In rational number operation “multiplication” is :

(A) Commutative (B) Associative (C) Both A and B (D) None of these

10. The additive inverse of a rational number $\frac{x}{y}$ is :

(A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (C) $-\frac{x}{y}$ (D) $-\frac{y}{x}$

11. Which rational numbers are equal to their reciprocals ?

(A) 0, 1 (B) – 1, 0 (C) –1, 1 (D) –1, 0, 1

12. If we multiply $\frac{4}{7}$ by the reciprocal of $\frac{1}{63}$, we get

(A) 36 (B) 34 (C) 28 (D) 63

13. A rectangular sheet of paper is 5 $\frac{1}{2}$ cm long and 10 $\frac{2}{3}$ cm wide. Perimeter of its rectangle is :

(A) $\frac{57}{3}$ cm (B) $\frac{47}{3}$ cm (C) $\frac{37}{3}$ cm (D) $\frac{97}{3}$ cm

14. Two rational numbers between $\frac{11}{15}$ and $\frac{17}{20}$ is :

(A) $\frac{3}{4}, \frac{5}{6}$ (B) $\frac{3}{5}, \frac{1}{2}$ (C) $\frac{1}{2}, \frac{5}{7}$ (D) $\frac{5}{7}, \frac{6}{7}$

15. Which rational number is exactly in middle between the two rational numbers $\frac{1}{3}$ & $\frac{1}{2}$?

(A) $\frac{5}{12}$ (B) $\frac{5}{6}$ (C) $\frac{1}{12}$ (D) $\frac{3}{5}$

16. How many rational numbers exist between any two distinct rational numbers ?

(A) 2 (B) 3 (C) 11 (D) Infinite numbers

17. Choose the rational number which does not lie between rational numbers $\frac{3}{5}$ and $\frac{2}{3}$.

(A) $\frac{46}{75}$ (B) $\frac{47}{75}$ (C) $\frac{49}{75}$ (D) $\frac{50}{75}$

FILL IN THE BLANKS

1. When the numerator and denominator of a rational number have _____ sign, then rational number is positive.

2. _____ is the only rational number whose negative is the number itself.

3. A rational number is said to be in standard form if denominator is_____ and there is no _____ factor between numerator and denominator.

4. Find x if $\frac{-1}{5} = \frac{8}{x}$

5. The rational number(s) which is/are reciprocal of itself _____

6. Product of $-1\frac{1}{7}$ and reciprocal of $\frac{-2}{7}$ is _____

TRUE / FALSE

1. 0 is neither positive nor negative

2. Every positive rational number is greater than every negative rational number.

3. All integers are rational numbers.

4. $\frac{-3}{5}$ lies to the left of 0 on the number line.

5. $\left[\frac{-2}{7} - \frac{5}{7}\right] \times \left[\frac{8}{3} \div \frac{4}{9}\right] = -6$

6. When we divided the difference of $\frac{3}{7}$ and $\frac{2}{5}$ by the product of $\frac{4}{5}$ and $\frac{25}{2}$, we get $\frac{2}{7}$

7. There exist finite numbers between two given rational numbers.

MATCH THE COLUMNS

1.

	Column – I		**Column – II**
(A)	$\frac{5}{6} \times \left(\frac{-4}{5} + \frac{-7}{10}\right) = \left(\frac{5}{6} \times \frac{-4}{5}\right) + \left(\frac{5}{6} \times \frac{-7}{10}\right)$	(p)	Closure law
(B)	$\frac{3}{8} + \frac{-5}{6} = \frac{-5}{6} + \frac{3}{8}$	(q)	Associative law
(C)	The sum of two rational numbers is a rational number	(r)	Distributive law
(D)	$\frac{2}{3} \times \left(\frac{6}{7} \times \frac{-14}{15}\right) = \left(\frac{2}{3} \times \frac{6}{7}\right) \times \frac{-14}{15}$	(s)	Commutative law
(E)	Reciprocal of $\frac{-14}{17}$	(t)	$\frac{17}{14}$
(F)	Additive inverse of $\frac{-17}{14}$	(u)	$\frac{17}{-14}$

SECTION –B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. For what value of 'a' the number $\frac{-11}{a}$ is not a rational number

2. In the standard form of a rational numbers, What is the common factor of numerator and denomerator?

3. Find the standard form of $\frac{-48}{60}$

4. What should be added to $\frac{2}{9}$ to get -1 ?

5. What should be subtracted from $\frac{-2}{3}$ to get $\frac{-5}{6}$?

6. Find $\frac{3}{4}$ of $5\frac{1}{3}$

7. Find the additive inverse of the following :

(a) $\frac{1}{3}$ **(b)** 0 **(c)** 5 **(d)** $\frac{5}{-7}$

8. Find the multiplicative inverse of the following :

(a) $\frac{7}{5}$ **(b)** 0 **(c)** – 8 **(d)** $\frac{-2}{5}$

SHORT ANSWER TYPE

9. Find four rational numbers equivalent to $\frac{-7}{13}$.

10. Find x such that $\frac{13}{6} = \frac{-65}{x}$

11. Express $\frac{-36}{24}$ as a rational number with numerator (i) –9 (ii) 6

12. $\frac{7}{12}$ of the total number of children in a school come by bus, $\frac{3}{10}$ walk and the rest of them come by car. If there are 1020 children in the school, find how many come

(a) by bus **(b)** on foot **(c)** by car

13. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

14. The cost of 2 $\frac{1}{2}$ metres of cloth is Rs $78\frac{3}{4}$. Find the cost of cloth per metre.

15. A man bought $4\frac{1}{2}$ kg of apples , $2\frac{3}{4}$ kg of guavas and $3\frac{1}{5}$ kg of oranges . How many kg of fruits did he buy?

16. Add $\frac{332}{30}$ to the difference of $3\frac{1}{2}$ and $1\frac{1}{6}$

17. A car is moving at an average speed of $40\frac{2}{5}$ km/hr. How much distance will it cover in $7\frac{1}{2}$ hours ?

LONG ANSWER TYPE

18. Arrange the following rational numbers $\frac{-4}{9}, \frac{5}{12}, \frac{-7}{18}, \frac{2}{-3}$ in descending order

19. Represent $\frac{-3}{4}$ and $\frac{3}{4}$ on the number line.

20. Find five rational number between $\frac{-3}{5}$ and $\frac{1}{2}$

21. Divide the sum of $\frac{-1}{3}$ and $\frac{5}{6}$ by the sum of $\frac{1}{-4}$ and $\frac{3}{8}$.

22. Simplify : $[2 \div (1+1 \div \frac{1}{2})]$ $[3 \div (\frac{5}{6} \text{of} \frac{3}{2} \div 1\frac{1}{4})]$

23. Find the value of $1 + \cfrac{1}{1+\cfrac{1}{1-\frac{1}{6}}}$

24. Find the product of $\left(1-\frac{1}{n}\right)\left(1-\frac{1}{n+1}\right)\left(1-\frac{1}{n+2}\right)$ ..... $\left(1-\frac{1}{2n}\right)$

EXERCISE 02

SECTION –A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

1. A body floats 2/9 of its volume above the surface. What is the ratio of the body submerged volume to its exposed volume?

(A) 7 : 2 (B) 9 : 2 (C) 9 : 7 (D) 7 : 9

2. When simplified the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)$ ..... $\left(1+\frac{1}{n}\right)$ becomes –

(A) n (B) $\frac{n-1}{2}$ (C) $\frac{n+1}{2}$ (D) $\frac{n}{2}$

3. Find the product of $\left(1-\frac{1}{6}\right)\left(1-\frac{1}{7}\right)\left(1-\frac{1}{8}\right)...\left(1-\frac{1}{n+4}\right)\left(1-\frac{1}{n+5}\right)$ for $n \geq 4$.

(A) $\frac{1}{n+5}$ (B) $\frac{5}{n+5}$ (C) $\frac{2}{n+4}$ (D) $\frac{3}{(n+4)(n+5)}$

4. The simplification of $7\frac{1}{2}-\left[2\frac{1}{4} \div \left\{1\frac{1}{4}-\frac{1}{2}\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right]$ yields.

(A) $\frac{2}{9}$ (B) $4\frac{1}{2}$ (C) $9\frac{1}{2}$ (D) $1\frac{77}{228}$

5. A student was asked to multiply a number by $\frac{3}{2}$. Instead he divided the number by $\frac{3}{2}$ and obtained a number smaller by $\frac{2}{3}$; the number is

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

6. If 1.5 x = 0.04 y, then the value of $\left(\frac{y-x}{y+x}\right)$ is :

(A) $\frac{730}{77}$ (B) $\frac{73}{77}$ (C) $\frac{7.3}{77}$ (D) None of these

7. Simplify : $4 - \cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\frac{1}{4}}}}$

(A) $\frac{23}{8}$ (B) $\frac{1}{8}$ (C) $\frac{23}{4}$ (D) None of these

8. If , $\frac{3}{7}+x+\left(\frac{-8}{21}\right)+\frac{5}{22}=\frac{2}{462}$ then find x

(A) $\frac{125}{462}$ (B) $\frac{-125}{462}$ (C) $\frac{25}{462}$ (D) none of these

9. One day at Central High School. $\frac{1}{12}$ of the students were absent, and $\frac{1}{5}$ of those present went on a field trip. If the number of students staying in school was 704, how many students are enrolled at Central High School ?

(A) 960 (B) 900 (C) 600 (D) 660

SECTION –B (TECHIE STUFF)

10. Which of the following is an irrational number :

(A) 3.65789.... (B) 3.65765..... (C) 3.65786578..... (D) 3.6666.....

11. An irrational number between 3 and 5 is :

(A) $\sqrt{6}$ (B) $\sqrt{15}$ (C) $\sqrt{7}$ (D) $\sqrt{30}$

EXERCISE 03

PREVIOUS YEAR EXAMINATION QUESTIONS

1. Which property is shown below in equation below? **[NSTSE 2009]**

$\frac{-7}{5}+(\frac{2}{-11}+\frac{-13}{25})=(\frac{-7}{5}+\frac{2}{-11})+\frac{-13}{25}$

(A) closure (B) commutative (C) associative (D) identity

2. Which pair of operation will make the equation below true when inserted into the blank spaces in the order shown ?

$2\frac{3}{10}$ ___ 1.5 ___2 = 1.8 **[NSTSE 2009)**

(A) – and + (B) and + (C) + and – (D) and –

3. Which one of the following is the rational number lying between $\frac{6}{7}$ and $\frac{7}{8}$? **[NSTSE 2010]**

(A) $\frac{3}{4}$ (B) $\frac{99}{112}$ (C) $\frac{95}{112}$ (D) $\frac{97}{112}$

4. Shalini shaded 3/5 of her design given on the right.

Which decimal number represents the shaded part of her design? **[IMO–2010]**

(A) 0.6 (B) 0.06 (C) 0.006 (D) 0.0006

5. The fractions $\frac{2}{4},\frac{3}{6},\frac{4}{8}$ and $\frac{5}{10}$ are each equivalent to $\frac{1}{2}$. Which of the following statements represents the relation between numerator and denominator of the fractions given above?

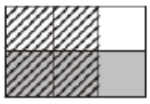
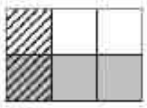
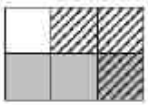
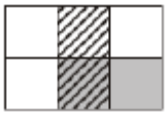
(A) The numerator is twice the denominator **[IMO–2010]**

(B) The denominator is twice the numerator

(C) The numerator is 2 more than the denominator

(D) The denominator is 2 more than the numerator

6. Which model best represents the expression $\frac{1}{2}\times\frac{2}{3}$? **[IMO–2010]**

(A)  (B)  (C)  (D) 

7. What number should be added to $\frac{-7}{10}$ to get $\frac{-5}{16}$? **[NSTSE 2011]**

(A) $\frac{-31}{80}$ (B) $\frac{13}{80}$ (C) $\frac{131}{80}$ (D) $\frac{31}{80}$

8. In the following rational numbers, x is a positive integer. Which among these is the least ? **[NSTSE 2012]**

(A) $\frac{-x}{3}$ (B) $\frac{-x}{4}$ (C) $\frac{x}{5}$ (D) $\frac{-x}{6}$

9. If $\frac{x}{y}=\frac{3}{4}$, then the value of $\left(\frac{6}{7}+\frac{y-x}{y+x}\right)$ equals_____. **[IMO–2012]**

(A) $\frac{5}{7}$ (B) $1\frac{1}{7}$ (C) 1 (D) 2

10. Which of the following will make the expression ture? **[IMO–2013]**

$$\left[\frac{7}{3}-\frac{2}{6}\right]-\frac{1}{2}+\left[\frac{4}{5}\div\frac{6}{8}\right]+\frac{3}{4}\square\frac{4}{5}\div\left(\frac{6}{8}+\frac{3}{4}\right)-\left(\frac{3}{4}+\frac{5}{6}\right)+1\frac{7}{3}$$

(A) > (B) < (C) = (D) Can't be determined

11. Arrange the following numbers in the ascending order. **[IMO–2013]**

$\frac{1}{4},\frac{6}{7},\frac{-3}{5},\frac{-5}{10},\frac{-2}{7}$

(A) $\frac{-5}{10},\frac{-2}{7},\frac{-3}{5},\frac{1}{4},\frac{6}{7}$ (B) $\frac{-3}{5},\frac{-5}{10},\frac{-2}{7},\frac{1}{4},\frac{6}{7}$ (C) $\frac{1}{4},\frac{6}{7},\frac{-5}{10},\frac{-2}{7},\frac{-3}{5}$ (D) $\frac{-2}{7},\frac{-3}{5},\frac{-5}{10},\frac{1}{4},\frac{6}{7}$

12. Which of the following sum is in the simplest form? **[IMO–2013]**

(A) $\frac{4}{9}+\frac{-5}{9}$ (B) $\frac{-5}{12}+\frac{11}{-12}$ (C) $\frac{-2}{5}+\frac{13}{20}$ (D) $\frac{-7}{8}+\frac{1}{12}+\frac{2}{3}$

13. The product of two rational number is $\frac{-8}{9}$. If one of the numbers is $\frac{-4}{15}$ what is the other number ? **[NSTSE 2014]**

(A) $3\frac{1}{3}$ (B) $\frac{3}{10}$ (C) $1\frac{1}{9}$ (D) $\frac{9}{10}$

14. If $\frac{3p+2}{5}-\frac{4p-3}{7}+\frac{p-1}{35}=4$, find the value of 'p'. **[NSTSE 2014]**

(A) 65 (B) 63 (C) 36 (D) 56

ANSWER KEY

EXERCISE 01

SECTION –A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	C	A	C	B	B	B	C	C	C	A	D	A	A
Ques.	16	17													
Ans.	D	D													

FILL IN THE BLANKS

1. Same **2.** Zero **3.** Positive , Common **4.** – 40

5. 1 and – 1 **6.** 4

TRUE / FALSE

1. True **2.** True **3.** True **4.** True **5.** True

6. False **7.** False

MATCH THE COLUMNS

1. (A) – (r), (B) – (s), (C) – (p), (D) – (q) , (E) – u (F) – t

SECTION –B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. 0 **2.** 1 **3.** $\frac{-4}{5}$ **4.** $\frac{-11}{9}$ **5.** $\frac{1}{6}$

6. 4

7. **(a)** $-\frac{1}{3}$ **(b)** 0 (c) – 5 **(d)** $\frac{5}{7}$

8. **(a)** $\frac{5}{7}$ **(b)** Not defined **(c)** $\frac{-1}{8}$ **(d)** $\frac{5}{-2}$

SHORT ANSWER TYPE

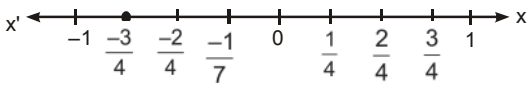
9. $\frac{-14}{26}, \frac{-21}{39}, \frac{-28}{52}, \frac{-35}{65}$ **10.** – 30 **11.** **(i)** $\frac{-9}{6}$ **(ii)** $\frac{6}{-4}$

12. **(a)** 595 **(b)** 306 **(c)** 119 **13.** $\frac{4}{3}$

14. Rs. $31\frac{1}{2}$ **15.** $\frac{209}{20}$ **16.** $\frac{67}{5}$ **17.** 303 km

LONG ANSWER TYPE

18. $\frac{5}{12} > \frac{-7}{18} > \frac{-4}{9} > \frac{2}{-3}$

19. 

20. $\frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{3}, \frac{1}{4}$

21. 4

22. 2

23. $\frac{16}{11}$

24. $\frac{n-1}{2n}$

EXERCISE 02

SECTION –A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9
Ans.	A	C	B	B	A	B	B	B	A

SECTION –B (TECHIE STUFF)

Ques.	10	11
Ans.	A	B

EXERCISE 03

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	C	C	D	A	B	A	D	A	C	A	B	A	A	D