# MATHEMATICS

## Class-VII

## Topic-05

## TRIANGLES

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CHAPTER 05

# TRIANGLES

## TERMINOLOGIES

**Polygon, Convex Polygon, Concave Polygon, Regular Polygon,Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Decagon, Isosceles triangle, Equilateral triangle, Scalene triangle, Acute triangle, Right triangle. Obtuse triangle, Interior of a triangle, Exterior of a triangle, Triangular region, Median, Altitude, Triangle inequality, Pythagoras theorem.**

## INTRODUCTION

In this chapter first we will first deal with polygon, and also the polygon with 3 sides i.e triangle.

'Tri' means three and the triangle means three angles. Triangle is a simple closed figure made up of three line segments (on joining three non collinear points), or we can say a triangle is a figure made up by three line segments joining, in pairs, three non-collinear points. That is, if A, B, C are three non-collinear points, the figure formed by three line segments AB, BC and CA is called a triangle with vertices A, B, C.

## 5.1 TRIANGLES

### (a) Polygon

A closed plane figure bounded by line segments is called a **polygon**.

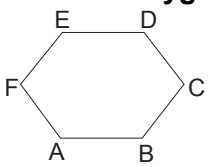
A polygon is named according to the number of sides it has :

| **No. of sides** | 3 | 4 | 5 | 6 | 7 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| **Figure** | Triangle | Quadrilateral | Pentagon | Hexagon | Heptagon | Octagon | Decagon |

In general, a polygon having **n** sides is called 'n' sided polygon.

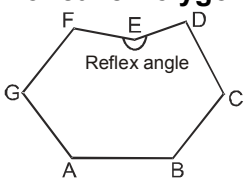
**Diagonal of Polygon** :
Line segment joining any two non-consecutive vertices of a polygon is called its **diagonal**.

**Convex Polygon** :




If all the interior angles of a polygon are less than $180^0$, it is called a **convex polygon**.

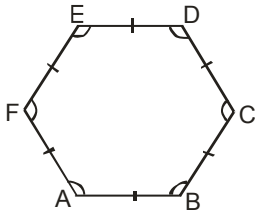
**Concave Polygon :**



If one or more of the interior angles of a polygon is greater than $180^0$ i.e. reflex, it is called a **concave polygon**.

**Regular Polygon :**

A polygon is called a **regular polygon** if all its sides have equal length and all the angles have equal measure.




## NOTE :

**(i)** The sum of the interior angles of a convex polygon of n sides is (2n – 4) right angles or (2n – 4) 90º.

**(ii)** The sum of the **exterior angles** of a convex polygon is 4 right angles or 360º.

**(iii)** Each interior angle of a n-sided regular polygon is $\frac{(2n-4)\times 90^\circ}{n}$.

**(iv)** Each exterior angle of a regular polygon of n sides $= \left(\frac{360^0}{n}\right)$.

**(v)** If a polygon has n sides, then the number of diagonals of the polygon = $\frac{n(n-3)}{2}$.

**Illustration 5.1**

If the sum of interior angles of a polygon is 1620º, find its number of sides.

**Sol.** We know that sum of all interior angles of a n sided polygon = (2n – 4) right angles

$= [(2n-4)\times 90^0]$

but given sum of interior angles = 1620º

$(2n-4)\times 90^0 = 1620^\circ$

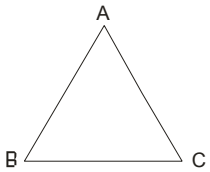
$\Rightarrow \quad 2n-4 = \frac{1620^\circ}{90^\circ}$

$\Rightarrow \quad 2n-4 = 18$

$\Rightarrow \quad 2n = 22$

$\Rightarrow \quad n = 11.$

## (b) Some definations and concepts related to triangles




**Sides :** The three line segments forming a triangle are called the sides of the triangle. [AB, BC, CA are the sides]

**Elements or Parts :** The three sides and three angles of a triangle are together called the six parts or elements of the triangle.

**Types of Triangles :**

**Isosceles triangle :** A triangle whose two sides are equal, is called an **isosceles triangle**.

**Equilateral triangle :** A triangle whose all sides are equal, is called an **equilateral triangle**.

**Scalene triangle :** A triangle whose no two sides are equal, is called a **scalene triangle**.

**Acute triangle :** A triangle whose all the angles are acute is called an **acute triangle**.

**Right triangle :** A triangle whose one of the angles is a right angle is called a **right triangle**.

**Obtuse triangle :** A triangle whose one of the angles is an obtuse angle is called an **obtuse triangle**.

**Interior of a triangle :** The interior of a triangle is made up of all such points on the plane, that are enclosed by the triangle.

**Exterior of a triangle :** The exterior of a triangle is that part of the plane which consists of those points, which are neither on the triangle nor in its interior.

**Triangular region :** The interior of a triangle together with the triangle itself is called the **triangular region**.

**Median :** A **median** connects a vertex of a triangle to the mid-point of the opposite side. A triangle has 3 median. The pont of intersection of all the median of triangle is known as **Centroid**

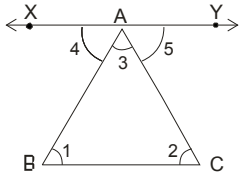
**Altitude :** A perpendicular line segment drawn from a vertex of a triangle to the opposite side is known as **altitude**. A triangle has 3 altitudes.The point of intersection of all the three altitude of triangle is known as **Orthocentre**

**Angle bisector:** The line drawn from vertex of the triangle which bisect the angle of triangle is known as angle bisector and the point at which all angle bisector of triangle meets is known **Incentre**

**Circumcentre :** The point of intersection of perpendicular bisector of all side of triangle is known as **Circumcentre**

**(i) Property :** The sum of the angles of a triangle is two right angles or 180°.

**Proof :** Let ABC be any triangle. Through A, draw a line XY parallel to the side BC as shown in figure. The angles are shown in figure.




Since, XY || BC and the transversal AB cuts XY and BC at A and B respectively.

$\therefore \quad \angle 1 = \angle 4$ [$\because$ Alternate interior angles are equal]

Similarly, XY || BC and the transversal AC cut XY and BC at A and C respectively.

$\therefore \quad \angle 2 = \angle 5$ [$\because$ Alternate interior angles are equal]

Also, $\angle 3 = \angle 3$

Adding the angles on the respective sides, we get

$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 3$

But, $\angle 4 + \angle 5 + \angle 3 = 180^o$

$\therefore \quad \angle 1 + \angle 2 + \angle 3 = 180^o$ = 2 right angles

Hence, the sum of the angles of a triangle is two right angles is $180^o$.

**Some Important Results :**

From the above property, we obtain the following useful results :

**(i)** A triangles cannot have more than one right angle.

**(ii)** A triangles cannot have more than one obtuse angle i.e. if one angle of a triangle is obtuse then the other two are acute.

**(iii)** In a right triangle, the other two angles are acute and their sum is 90º.

**Illustration 5.2**

Two angles of a triangle are of measures 75º and 35º. Find the measure of the third angle.

**Sol.** Let ABC be a triangle such that $\angle B = 75^\circ$ and $\angle C = 35^\circ$. Then, we have to find the measure of the third angle A.

Now, $\angle B = 75^\circ$ and $\angle C = 35^\circ$

$\Rightarrow$ $\angle B + \angle C = 75^\circ + 35^\circ = 110^\circ$

By the angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $\angle A + 110^\circ = 180^\circ$ $[\because \angle B + \angle C = 110^\circ]$

$\Rightarrow$ $\angle A = 180^\circ - 110^\circ$

$\Rightarrow$ $\angle A = 70^\circ$.

**Illustration 5.3**

Out of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.

**Sol.** Let the smallest angle of the given triangle be of $x^\circ$. Then, the other two angles are of measures $2x^\circ$ and $3x^\circ$

$\therefore$ $x + 2x + 3x = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow$ $6x = 180^\circ$ $\Rightarrow$ $x = \frac{180^\circ}{6} = 30^\circ$.

Hence, the angles of the triangle are 30º, 60º and 90º.

**Illustration 5.4**

If the angles of a triangle are in the ratio 2 : 3 : 4, determine three angles.

**Sol.** Let measures of the angles of triangle be $2x^\circ$, $3x^\circ$ and $4x^\circ$ . Then,

$2x + 3x + 4x = 180^\circ$ [Sum of the angles of triangle is 180º]

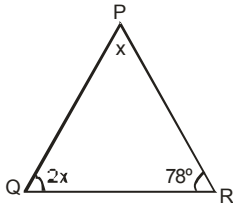
$\Rightarrow$ $9x = 180^\circ$

$\Rightarrow$ $x = 20^\circ$

Hence, the angles of the triangle are 40º, 60º, 80º.

**Illustration 5.5**

Find all the angles of triangle in the given figure.



**Sol.** In $\Delta PQR$, by angle sum property of a triangle,

$\angle P + \angle Q + \angle R = 180^\circ$

$\Rightarrow$ $x + 2x^\circ + 78^\circ = 180^\circ$

$\Rightarrow$ $3x + 78^\circ = 180^\circ$

$\Rightarrow$ $3x = 180^\circ - 78^\circ$

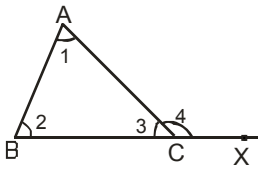
$\Rightarrow$ $3x = 102^\circ$

$\Rightarrow$ $x = 34^\circ$

$\Rightarrow$ $\angle P = x = 34^\circ$, $\angle Q = 2x = 2 \times 34^\circ = 68^\circ$ and $\angle R = 78^\circ$.

**(ii) Property :** If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the interior opposite angles.

**Proof :** Let ABC be a triangle such that its side BC is produced to form ray BX. Then, $\angle ACX = \angle 4$ is the exterior angle of $\Delta ABC$ at C and $\angle 1$ and $\angle 2$ are two interior opposite angles.




We have to prove that

$\angle 4 = \angle 1 + \angle 2$

The sum of the angles of a triangle is $180^\circ$

$\therefore$ $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ . ...(i)

Also, $\angle 3 + \angle 4 = 180^\circ$ ...(ii)

[$\because$ $\angle 3$ and $\angle 4$ form a linear pair]

From (i) and (ii), we get

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow$ $\angle 1 + \angle 2 = \angle 4$ [Subtracting $\angle 3$ from both sides]

Hence, $\angle 4 = \angle 1 + \angle 2$.

**(iii) Property :** In any triangle, an exterior angle is greater than either of the interior opposite angles.

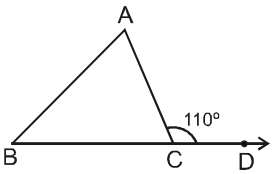
From the above proof, $\angle ACX = \angle A + \angle B$

$\Rightarrow$ $\angle ACX > \angle A$ & $\angle ACX > \angle B$

**Illustration 5.6**

An exterior angle of a triangle is $110^\circ$, and one of the interior opposite angles is $30^\circ$. Find the other two angles of the triangle.

**Sol.** Let ABC be a triangle whose side BC is produced to D to form an exterior angle $\angle ACD$ such that $\angle ACD = 110^\circ$.




Let $\angle B = 30^\circ$. By exterior angle theorem, we have

$\angle ACD = \angle B + \angle A$

$\Rightarrow$ $110^\circ = 30^\circ + \angle A$

$\Rightarrow$ $\angle A = 110^\circ - 30^\circ = 80^\circ$

Now, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $80^\circ + 30^\circ + \angle C = 180^\circ$

$\Rightarrow$ $\angle C = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$.

**Illustration 5.7**

One of the exterior angles of a triangle is $80^\circ$ and the interior opposite angles are in the ratio 3 : 5. Find the angles of the triangle.

**Sol.** Let $\angle ACX$ be the exterior angle of $\Delta ABC$ at C such that $\angle ACX = 80^\circ$. Clearly, $\angle A$ and $\angle B$ are the interior opposite angles.

It is given that $\angle A : \angle B = 3 : 5$. So, let $\angle A = 3x^\circ$ and $\angle B = 5x^\circ$.

$\angle ACX = \angle A + \angle B$

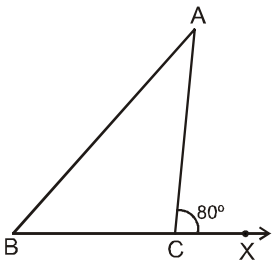
$\Rightarrow$ $80^\circ = 3x + 5x$ [By exterior angle property]

$\Rightarrow$ $8x = 80^\circ$

$\Rightarrow$ $\frac{8x}{8} = \frac{80^\circ}{8}$ [Dividing both sides by 8]

$\Rightarrow$ $x = 10^\circ$

$\therefore$ $\angle A = 3x^\circ = 30^\circ$ and $\angle B = 5x^\circ = 50^\circ$




Now, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $30^\circ + 50^\circ + \angle C = 180^\circ$

$\Rightarrow$ $80^\circ + \angle C = 180^\circ$

$\Rightarrow$ $\angle C = 180^\circ - 80^\circ = 100^\circ$

Hence, $\angle A = 30^\circ$, $\angle B = 50^\circ$ and $\angle C = 100^\circ$

**Alternatively :**

Because $\angle ACB$ and $\angle ACX$ form linear pair.

So, $\angle ACB + \angle ACX = 180^\circ$

$\Rightarrow$ $\angle ACB + 80^\circ = 180^\circ$ [$\because \angle ACX = 80^\circ$]

$\Rightarrow$ $\angle ACB = 100^\circ$

Now, if $\angle A$ & $\angle B$ are in ratio of 3 : 5.

Let $\angle A = 3x$, $\angle B = 5x$.

Now, from $\Delta ABC$

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $3x + 5x + 100^\circ = 180^\circ$

$\Rightarrow$ $8x = 180^\circ - 100^\circ$

$\Rightarrow$ $x = 10^\circ$.

$\angle A = 3x = 3 \times 10^\circ = 30^\circ$, $\angle B = 5x = 5 \times 10^\circ = 50^\circ$.

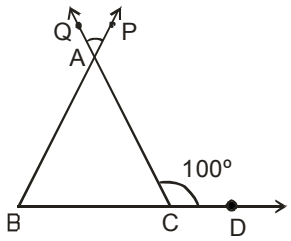
**Illustration 5.8**

Sides BC, CA and BA of triangle ABC are produced to D, Q, P respectively as shown in figure. If ∠ACD = 100º and ∠QAP = 35º, find all the angles of the triangle.

**Sol.** Since ∠QAP and ∠BAC are vertically opposite angles.

∴ ∠BAC = ∠QAP

⇒ ∠BAC = 35º [∵ ∠QAP = 35º]




By exterior angle theorem, we have

∠ACD = ∠BAC + ∠CBA

⇒ 100º = 35º + ∠CBA

⇒ ∠CBA = 100º – 35º

⇒ ∠CBA = 65º.

Since, ∠ACB and ∠ACD form linear pairs.

∴ ∠ACB + ∠ACD = 180º

⇒ ∠ACB + 100º = 180º

⇒ ∠ACB = 180º – 100º = 80º

Hence, the angles of the ΔABC are ∠A = 35º, ∠B = 65º and ∠C = 80º.

**(iv) Property :** Angles opposite to equal sides of a triangle are equal.

**(v) Property :** Sides opposite to equal angles of a triangle are equal.

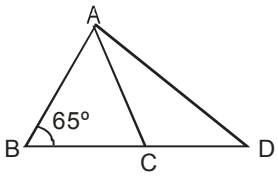
## Ask yourself

**1.** Find the angle of triangle which are in the ratio 3:4:5.

**2.** Two angle of a triangle are equal and the third angle measure 70º. Find the measure of each of the unknown angles.

**3.** If the sides of a triangle are produced in order, prove that the sum of exterior angle so formed is 360º.

**4.** In the figure below AB = AC = CD, the measure is ∠DAB




**5.** An exterior angle of triangle is 100 and its interior opposite angles in the ratio 2:3. Find the angles of the triangle.

**Answers**

**1.** 45°, 60°, 75° **2.** 55°, 55° **4.** 82.5° **5.** 40°, 60°, 80°

## 5.2 TRIANGLE INEQUALITY AND PYTHAGORAS THEOREM

### (a) Triangle Inequality

**Property :** The sum of any two sides of a triangle is greater than the third side.

**Illustration 5.9**

In each of the following there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle.
**(i)** 2, 3, 4 **(ii)** 2.5, 1.5, 4

**Sol.** **(i)** We have,
2 + 3 > 4, 2 + 4 > 3 and 3 + 4 > 2
That is, the sum of any two of the given numbers is greater than the third number.
So, 2 cm, 3 cm and 4 cm can be the lengths of the sides of a triangle.
**(ii)** We have,
2.5 + 1.5 $\not>$ 4.
So, the given numbers cannot be the lengths of the sides of a triangle.

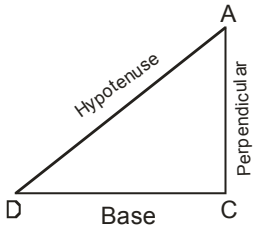
**Illustration 5.10**

The length of two sides of a triangle are 12 cm and 15 cm. Between what two measure should the length of the third side fall ?

**Sol.** Let x cm be the length of the third side. Then 12 + x > 15; 15 + x > 12 and 12 + 15 > x.
$\Rightarrow$ x > 15 – 12; x > 12 –15 and 27 > x $\Rightarrow$ x > 3 ; x > – 3 and 27 > x.
A number greater than 3 is obviously greater than – 3.
$\therefore$ x > 3 and 27 > x.
Hence, x lies between 3 cm and 27 cm.

### (b) Pythagoras Theorem

In a right triangle, if a, b are the lengths of the sides and c that of the hypotenuse, then $c^2 = a^2 + b^2$. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$




### (c) Converse of Pythagoras Theorem :

If the sides of a triangle are of lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is right angled and the side of length c is the hypotenuse.

### NOTE:

Three positive numbers a, b, c in this order are said to form a pythagorean triplet, if $c^2 = a^2 + b^2$. Triplets (3, 4, 5) (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37) are some pythagorean triplets.

**Illustration 5.11**

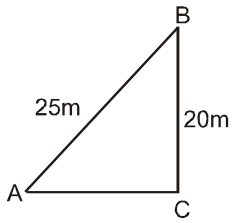
The sides of certain triangles are given below. Determine which of them are right triangles :
**(i)** a = 6 cm, b = 8 cm and c = 10 cm
**(ii)** a = 5 cm, b = 8 cm and c = 11 cm.

**Sol.** **(i)** Here the larger side is c = 10 cm.
We have : $a^2 + b^2 = 6^2 + 8^2 = 36 + 64 = 100 = c^2$.
So, the triangle with the given sides is a right triangle.
**(ii)** Here, the larger side is c = 11 cm
Clearly, $a^2 + b^2 = 25 + 64 = 89 \neq c^2$.
So, the triangle with the given sides is not a right triangle.

**Illustration 5.12**

A ladder 25 m long reaches a window of a building 20 m above the ground. Determine the distance of the foot of the ladder from the building.

**Sol.** Suppose that AB is the ladder, B is the window and CB is the building. Then, triangle ABC is a right triangle, with right angle at C.




$\therefore$ $AB^2 = AC^2 + BC^2$
$\Rightarrow$ $25^2 = AC^2 + 20^2$
$\Rightarrow$ $AC^2 = 625 - 400 = 225$
$\Rightarrow$ AC = $\sqrt{225}$ m = 15 m.

Ask yourself

**1.** Can we form a triangle with sides 1,2,3

**2.** Two sides of triangle are 5cm and 9 cm long . What can be length of its third side?

**3.** The hypotenuse of a right triangle is 13 cm long. If one of the remaining two sides is of the length 5 cm, find the length of other side.

**4.** The sides of triangle are 11 cm,60 cm ,61 cm. Verify that it is right angled triangle.

**5.** If the hypotenuse of a right angled triangle is 41 cm and the area of the triangle is 180 sq cm, then Calculate the difference between the lengths of the legs of the triangle :

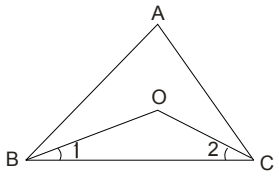
**Answers**

**1.** No **2.** 4 < third side < 14 **3.** 12 cm **5.** 31 cm

## Add your knowledge

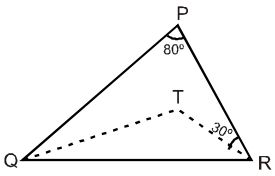
**Lets learn some important theorems which will make problems look easier:**

**Theorem** 1 : If the bisectors of angles ∠ABC and ∠ACB of a triangle ABC meet at a point O, then ∠BOC = 90° + $\frac{1}{2}$∠A.




**Example :**

In figure, TQ and TR are the bisectors of ∠Q and ∠R respectively. If ∠QPR = 80° and ∠PRT = 30°, determine ∠TQR and ∠QTR.




**Sol.** Since the bisectors of ∠Q and ∠R meet at T.

$\therefore$ $\quad \angle QTR = 90° + \frac{1}{2}\angle QPR$

$\Rightarrow$ $\quad \angle QTR = 90° + \frac{1}{2}(80°)$

$\Rightarrow$ $\quad \angle QTR = 90° + 40° = 130°$

In ΔQTR, we have

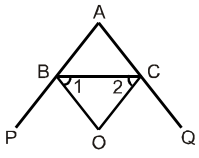
$\quad \angle TQR + \angle QTR + \angle TRQ = 180°$

$\Rightarrow$ $\quad TQR + 130° + 30° = 180°$ [$\because \angle TRQ = \angle PRT = 30°$ ]

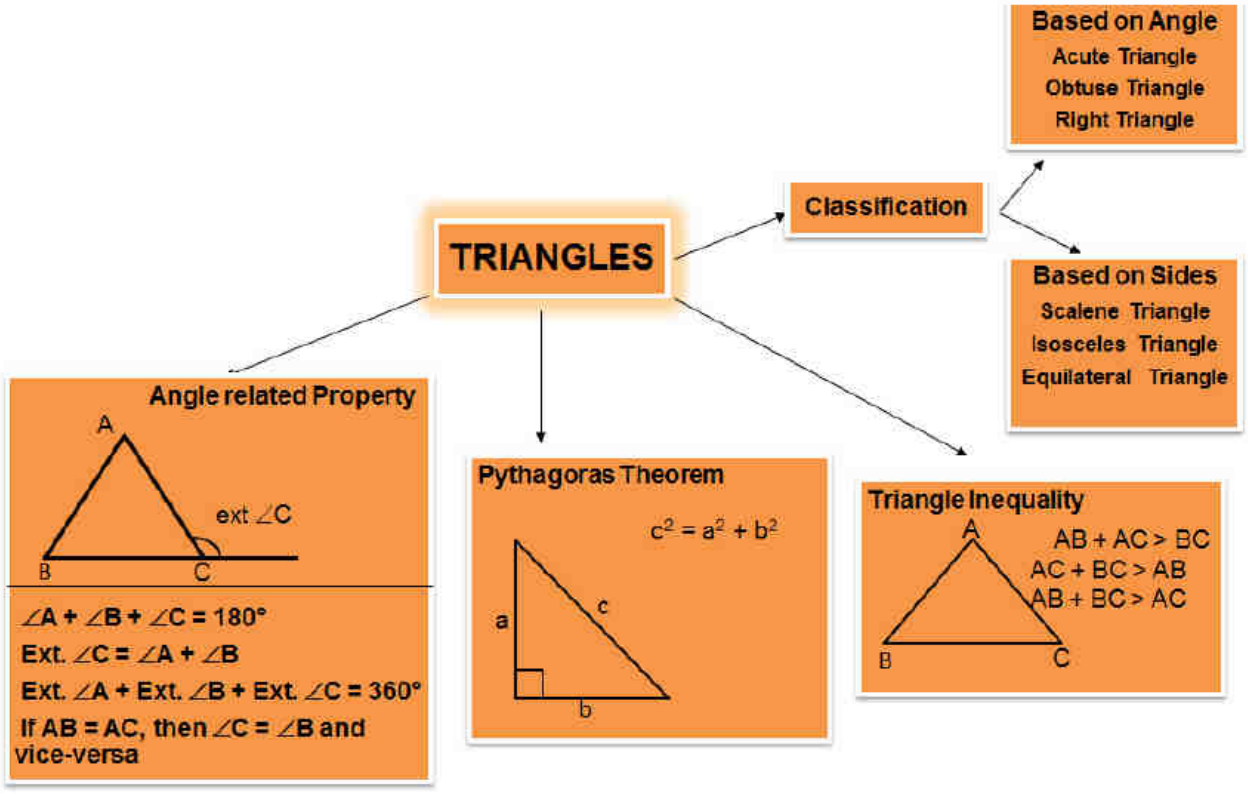
$\Rightarrow$ $\quad \angle TQR = 20°$

Thus, ∠TQR = 20° and ∠QTR = 130°.

**Theorem 2 :** The sides AB and AC of a ΔABC are produced to P and Q respectively. If the bisectors of ∠PBC and ∠QCB intersect at O, then ∠BOC = 90° – $\frac{1}{2}$∠A.



## Concept Map


TRIANGLES
Classification
Based on Angle
Acute Triangle
Obtuse Triangle
Right Triangle
Based on Sides
Scalene Triangle
Isosceles Triangle
Equilateral Triangle
Angle related Property
A
ext ∠C
B
C
∠A + ∠B + ∠C = 180°
Ext. ∠C = ∠A + ∠B
Ext. ∠A + Ext. ∠B + Ext. ∠C = 360°
If AB = AC, then ∠C = ∠B and vice-versa
Pythagoras Theorem
$c^2 = a^2 + b^2$
a
c
b
Triangle Inequality
A
B
C
AB + AC > BC
AC + BC > AB
AB + BC > AC

## Summary

**1.** The six elements of the triangle are its three angles and the three sides.

**(i)** The sum of the interior angles of a convex polygon of n sides is (2n – 4) right angles or (2n – 4) 90º.

**(ii)** The sum of the **exterior angles** of a convex polygon is 4 right angles or 360º.

**(iii)** Each interior angle of a n-sided regular polygon is $\frac{(2n-4)\times 90^{o}}{n}$.

**(iv)** Each exterior angle of a regular polygon of n sides = $\left(\frac{360^{0}}{n}\right)$.

**(v)** If a polygon has n sides, then the number of diagonals of the polygon = $\frac{n(n-3)}{2}$.

**2.** The line segment joining a vertex of a triangle to the mid point of its opposite side is called a median of the triangle. A triangle has 3 medians.

**3.** An exterior angle of a triangle is formed , when a side of a triangle is produced.

**4.** If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the interior opposite angles.

**5.** The sum of the angles of a triangle is two right angles or 180°.

**6.** A triangles cannot have more than one right angle.

**7.** Triangles cannot have more than one obtuse angle i.e. if one angle of a triangle is obtuse then the other two are acute.

**8.** In a right triangle, the other two angles are acute and their sum is 90º.

**9.** The sum of any two sides of a triangle is greater than the third side.

**10.** In a right triangle, the square of the hypotenuse equals the sum of the squares of the remaining two side. This is known as Pythagoras theorem.

**11.** In a right triangle, the hypotenuse is the longest side.

**12.** Of all the line segments that can be drawn to a given line from a point outside it, the perpendicular line segment is the shortest.

# EXERCISE 01

## SECTION -A (FIXED RESPONSE TYPE)

## MULTIPLE CHOICE QUESTIONS

**1.** Which of the following angles connot be an interior angle of any convex polygon ?
(A) 1º (B) 90º (C) 183º (D) 179º

**2.** Polygon in which sum of interior angles is equal to half the sum of exterior angles is a :
(A) Pentagon (B) Hexagon (C) Quadrilateral (D) None of these

**3.** If one of the interior angles of a regular polygon is to be equal to $\left(\frac{9}{8}\right)$ times of one of the interior angles of a regular hexagon, then the number of sides of the polygon is :
(A) 7 (B) 8 (C) 4 (D) 5

**4.** How many isosceles triangles are there with 40º as one of three angles ?
(A) 0 (B) 1 (C) 2 (D) 3

**5.** The measure of one of the angles of an isosceles triangle is 94º. Which of the following is definitely the measure of one of the other angles of the given triangle.
(A) 94º (B) 86º (C) 43º (D) 46º

**6.** Which of the following statements is false ?
(A) Each angle of an equilateral triangle is 60º.
(B) The angles at the base of an isosceles triangle are equal.
(C) Sides opposite to equal angles of a triangle may be unequal .
(D) The medians of an equilateral triangle are of equal length.

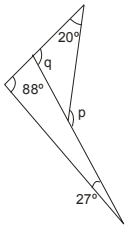
**7.** The angles of a triangle are in the ratio 3 : 2 : 1, then the angles are :
(A) 90º, 60º, 30º (B) 100º, 50º, 30º (C) 80º, 60º, 40º (D) 70º, 60º, 50º

**8.** If one angle of a triangle is equal to half the sum of the other two equal angles, the triangle is :
(A) Equilateral (B) Isosceles
(C) Right angled (D) Isosceles right angled

**9.** In a triangle, one angle is thrice the smallest angle and it is also greater than third angle by 23º, then greatest angle of triangle is :
(A) 64º (B) 81º (C) 87º (D) 92º

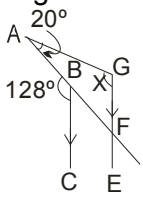
**10.** The vertical angles of an isosceles triangle measures (5p – 18)º and one of the base angles measures 3pº. What is the value of p ?
(A) 24 (B) 15 (C) 18 (D) 12

**11.** Find measure of p and q .



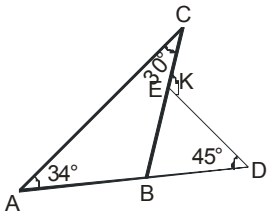

(A) p = 130°, q = 115°
(B) p = 115°, q = 135°
(C) p = 135°, q = 115°
(D) p = 135°, q = 110°

**12.** Find the value of X in the figure given below :



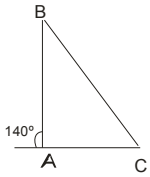

(A) 118°
(B) 18°
(C) 108°
(D) 128°

**13.** In the figure, K is equal to :




(A) 109°
(B) 112°
(C) 75°
(D) 79°

**14.** In the following diagram ∠B : ∠C = 3 : 4, find ∠C.

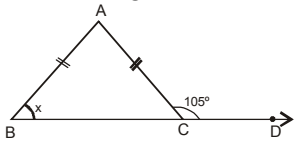



(A) 90°
(B) 100°
(C) 80°
(D) 60°

**15.** A ladder 17 m long, reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.
(A) 8 m
(B) 6 m
(C) 4 m
(D) 12 m

**16.** In ΔABC , AD bisects ∠BAC and AD= DC. If ∠ADB = 100°, then find ∠ABD.
(A) 30°
(B) 45°
(C) 60°
(D) 90°

**17.** In the figure given below, ABC is a triangle if AB = AC, then value of x is :




(A) 60°
(B) 70°
(C) 75°
(D) 105°

**18.** The number of triangles with any three of the lengths 1, 4, 6 and 8 cms consider is :
(A) 1
(B) 2
(C) 3
(D) 4

**19.** If three lengths are given a = 537 cm, b = 139 cm and c = 181 cm, then with these sides :
(A) One triangle can be constructed (B) No triangle can be constructed
(C) Two triangles can be constructed (D) None of these

**20.** For a triangle ABC which statement is always true :
(A) $AC^2 = AB^2 + BC^2$ (B) AC = AB + BC
(C) AC > AB + BC (D) AC < AB + BC

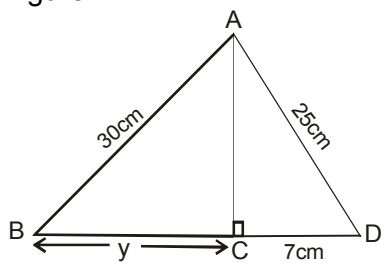
**21.** Which of the following is a pythagorean triplet ?
(A) 16, 18, 20 (B) 1, 2, 3 (C) 30, 40, 50 (D) 50, 51, 52

**22.** Which of the following can not be the measures of the sides of a triangle
(A) {15, 7, 8} (B) {3.5, 4.5, 5.5} (C) {3, 2.5, 1} (D) {5, 4, 4.5}

**23.** Pythagoras theorem is applied in
(A) acute angled triangle (B)obtuse angled triangle
(C) right angled triangle (D) none of these

**24.** Two sides of a triangle are 20 m and 40 m, find the maximum length of the third side, if angles are in the ratio of 1 : 2 : 3.
(A) 20.5 (B) 20 (C) 25 (D) 22

**25.** Find the length marked y in figure :




(A) 18 cm (B) 24 cm (C) 19 cm (D) 30 cm

**26.** In which of the following cases will it be possible to draw right angled triangle ?
(A) PQ = 12 cm, QR = 5cm, RP = 13 cm (B) m∠P = 90º, m∠Q = 45º, m∠R = 47º
(C) PQ = 12 cm, OR = 5 cm, RP = 14 cm (D) m∠P = 45º, m∠Q = 45º, m∠R = 120º

**27.** Which of the following is not the set of measures of the sides of a triangle ?
(A) 7, 3, 5 (B) 8, 12, 18 (C) 5, 6, 14 (D) 5, 12, 13

**28.** In which of the following cases can a right triangle ABC be constructed ?
(A) AB = 5 cm, BC = 7 cm, AC = 10 cm (B) AB = 7 cm, BC = 8 cm, AC = 12 cm
(C) AB = 8 cm, BC = 17 cm, AC = 15 cm (D) None of these

## FILL IN THE BLANKS

**1.** A polygon having six side is known as _____

**2.** A triangle cannot have more than ____ right angles.

**3.** A triangle whose no two sides are equal, is called a _____

**4.** Each angle of equilateral triangle is _____

**5.** A perpendicular line segment drawn from a vertex of a triangle to the opposite side is known as _____

**6.** An exterior angle is always _____ than either of the interior opposite angles .

**7.** The sum of any two sides of a triangle is always _____ than the third side.

**8.** In a right triangle, the _____ is the longest side

## TRUE / FALSE

**1.** Polygon with 8 sides is known as Decagon.

**2.** If the base angles of an isosceles triangle each measure 37 degrees, then the vertex angle has a measure of 106 degrees.

**3.** The altitude to the base of an isosceles traingle is also the median to the base.

**4.** The point of intersection of median of triangle is known as centroid.

**5.** Sum of the measure of the three angles of an acute triangle is less than the sum of the measure of the three angles of an obtuse triangle.

**6.** Traingle can be made from side 3cm , 6 cm, 9cm.

**7.** Triangle with side 8,6,10 form a right triangle

## MATCH THE COLUMN

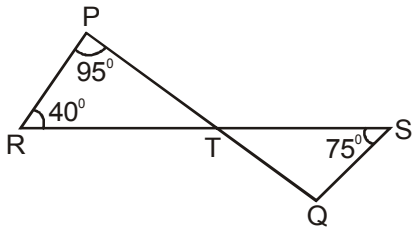
**1.**

| **Column – I** | | **Column – II** | |
|---|---|---|---|
| (A) | Polygon with all interior angles less than 180 degrees | (p) | $\left(\frac{360^0}{n}\right)$. |
| (B) | Each exterior angle of regular polygon with n sides | (q) | (2n – 4) 90º. |
| (C) | Sum of interior angle of convex polygon | (r) | convex |
| (D) | Each interior angle of regular convex polygon | (s) | $\frac{(2n-4)\ \times 90^\circ}{n}$. |
| (E) | number of diagonals of polygon | (t) | $\frac{n(n-3)}{2}$. |
| (F) | If two angles of triangle are 72° and 48°. Then 3rd angle is | (u) | 18° |
| (G) | one acute angle of right triangle is 72°. Find other acute | (v) | 60° |

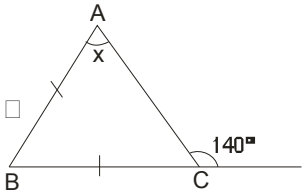
## SECTION -B (FREE RESPONSE TYPE)

### VERY SHORT ANSWER TYPE

**1.** In figure, lines PQ and RS intersect at point T. Such that ∠PRT = 40º, ∠RPT = 95º and ∠TSQ = 75º. Find ∠SQT = ?

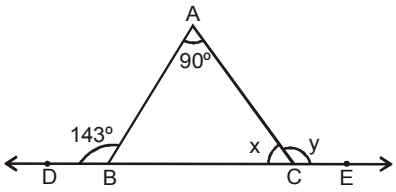



**2.** Find x, when AB = BC.

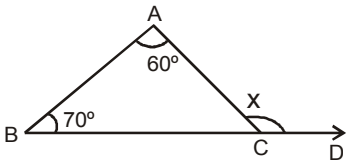



**3.** An exterior angle of a triangle is $100^0$ and one of the interior opposite angles is $35^0$. Find the other two angles of the triangle.

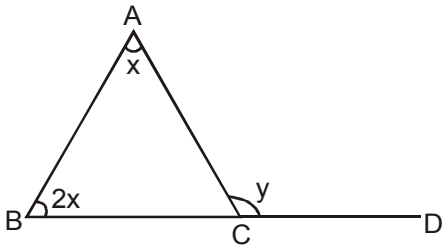
**4.** Find the value of x and y in the given figure.




**5.** Find the value of x in the given figure.




**6.** In figure AB = AC. Find x + y

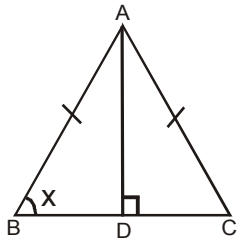



**7.** Is it possible to draw a triangle whose sides 5cm, 7 cm, 12 cm.

**8.** A man goes 24 due east and then 10 m due north. How far is he away from his intial position?

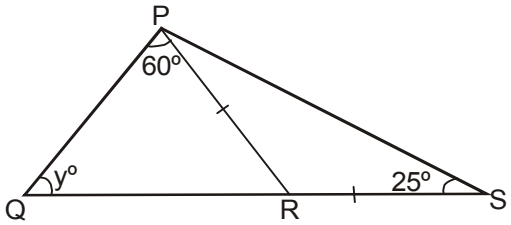
## SHORT ANSWER TYPE

**9.** Find x when $\angle CAD = 35^\circ$ and AB = AC also AD $\perp$ BC.

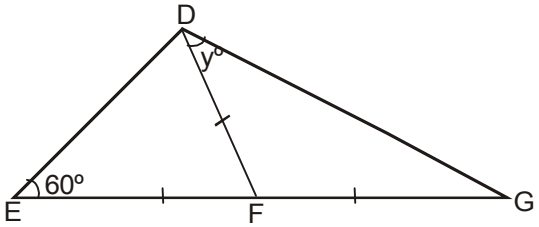



**10.** In each of the following figure, find the value of y.

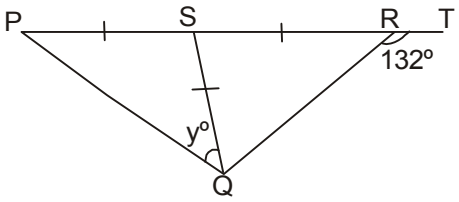
**(i)**



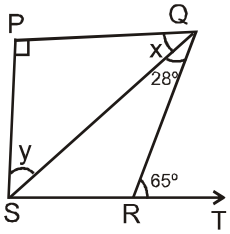

**(ii)**



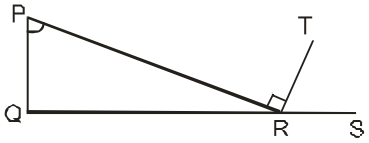

**(iii)**



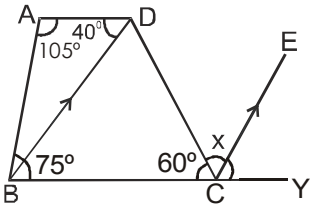

**11.** In figure, if PQ $\perp$ PS, PQ || SR, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the value of x & y.




**12.** In figure side QR of $\Delta$ PQR has been produced to S. If $\angle P : \angle Q : \angle R = 3 : 2 : 1$ and RT $\perp$ PR , find $\angle TRS$.




**13.** In figure, find value of x.



**14.** The length of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall.

**15.** Two poles of height 9 m and 14 m stand upright on a plane ground. If the distance between their feet is 12 m , find the distance between their tops.

**16.** ABC is an isosceles right triangle, right angle at C. Prove that $AB^2=2\ AC^2$

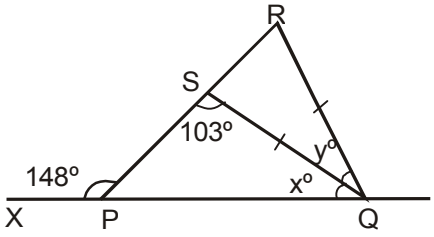
## LONG ANSWER TYPE

**17.** If there are 12 sides contained by a regular polygon, find the measure of each interior angle.

**18.** If each interior angle of a regular polygon is 120º, find the number of sides of the polygon.

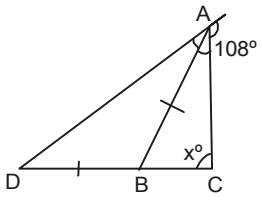
**19.** In $\Delta ABC$, $6\ \angle A = 4\ \angle B = 3\ \angle C$, find the angles of $\Delta ABC$

**20.** In given figure find the values of x and y. If QS = RQ.




**21.** The side BC of a $\Delta ABC$ is produced to D. If the bisector of $\angle A$ meets BC at point L. Prove that $\angle ABC + \angle ACD = 2\angle ALC$.

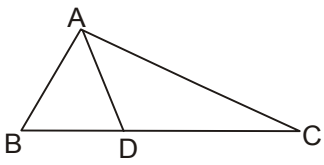
**22.** In given figure, AB divides $\angle DAC$ in the ratio of $\angle DAB : \angle BAC = 1 : 3$ and AB = DB. Find the value of x .




**23.** A tree is broken at a height of 5 meter from the ground and its top touches the ground at a distance of 12m from the base of the tree. Find original height of the tree.

**24.** In a quadrilateral ABCD, $\angle B = 90º$ if $AD^2 = AB^2 + BC^2 + CD^2$. Prove that $\angle ACD = 90º$.

**25.** In given figure, D is any point on the side BC of a triangle ABC. Show that AB + AC + BC > 2AD



# EXERCISE 02

## SECTION -A (COMPETITIVE EXAMINATION QUESTION)

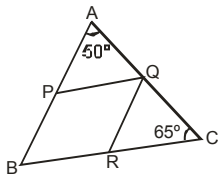
## MULTIPLE CHOICE QUESTIONS

**1.** In triangle PQR , if PQ= QR and $\angle Q = 100^0$ then $\angle R$ is equal to:
(A) 40º (B) 80º (C) 120º (D) 50º

**2.** In a $\Delta ABC$, $\angle B - \angle C = 22^0$ and $\angle C - \angle A = 7^0$. Find $\angle A$ of the triangle.
(A) 48º (B) 82º (C) 46º (D) 45º

**3.** One of the angle of a triangle is equal to the sum of the other two triangles. If the ratio of the other two angles is 4 : 5. Then the smallest angle is :
(A) 40º (B) 45º (C) 50º (D) 20º

**4.** In $\Delta ABC$, side AB || QR and side BC || PQ, $m\angle ACB = 65º$, $m\angle BAC = 50º$. Find $m\angle BPQ$.

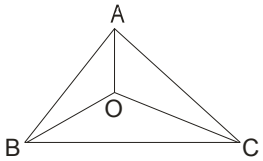



(A) 155º (B) 65º (C) 115º (D) 95º

**5.** The sum of two angles of a triangle is half of a right angle and their difference is 1º more than $\frac{2}{5}$ th of a right angle, then smallest angle of the triangle is :
(A) 4º (B) 41º (C) 40º (D) 3º

**6.** If in an isosceles triangle , each of the base angles is 40°, then the triangle is
(A) Right angled triangle (B) Acute angled triangle
(C) Obtuse angled triangle (D) Isosceles right angled triangle

**7.** O is a point in the interior of $\Delta$ ABC . State which of the following statement is true :




(A) OA + OB < AB (B) OB + OC < BC
(C) OA + OC = AC (D) AB + BC + AC < 2 ( OA + OB + OC )

**8.** The length of the three sides of a triangle are 6cm, 10 cm and x cm. Between what two whole number should the value of x lies ?
(A) 4 cm < x < 16 cm (B) 6 cm < x < 20 cm
(C) 3 cm < x < 10 cm (D) 2 cm < x < 8 cm

**9.** Two chimneys 18 m and 13 m high stand upright on a ground. If their feet is 12 m apart, then the distance between their tops is
(A) 5 m (B) 31 m (C) 13 m (D) 18 m

**10.** If the two legs of a right angled triangle are equal and the square of the hypotenuse is 100 $cm^2$, then the length of each leg is _____.

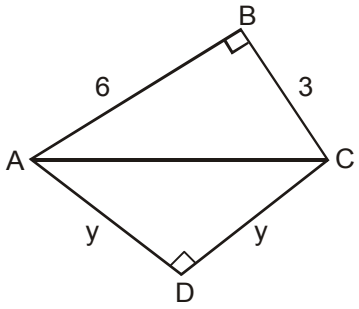
(A) 10 cm (B) $5\sqrt{2}$ cm (C) $10\sqrt{2}$ cm (D) $13\sqrt{2}$ cm

**11.** P and Q are the mid points of the sides AB and BC respectively of the triangle ABC, right-angled at B, then

(A) $AQ^2 + CP^2 = AC^2$ (B) $AQ^2 + CP^2 = \frac{4}{5}AC^2$

(C) $AQ^2 + CP^2 = \frac{5}{4}AC^2$ (D) $AQ^2 + CP^2 = \frac{3}{5}AC^2$

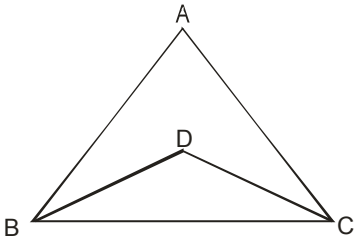
**12.** The value of y in the figure shown is




(A) $\frac{3}{\sqrt{5}}$ (B) $3\sqrt{2}$ (C) $3\sqrt{2.5}$ (D) 4.5

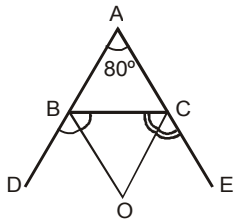
## SECTION -B (TECHIE STUFF)

**13.** In figure triangle ABC, BD and CD are angle bisectors. If $\angle ABC = 60°$ and $\angle BAC = 80°$. Find $\angle BDC$.




(A) 130° (B) 140° (C) 150° (D) None of these

**14.** In the figure $\angle A = 80º$, OB and OC are bisectors of $\angle DBC$ and $\angle ECB$ respectively, then find the measure of $\angle BOC$




(A) 40° (B) 50° (C) 60° (D) 30°

# EXERCISE 03

## PREVIOUS YEAR EXAMINATION QUESTIONS

**1.** Which of the following may lie outside or on the triangle? **[NSTSE 2009]**

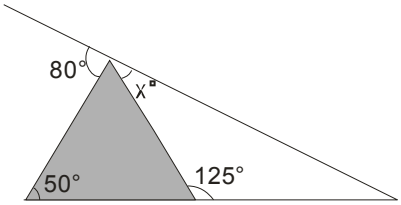
(i) circumcenter (ii) centroid (iii) orthocenter (iv) incenter

(A) i , ii, iii only (B) i and ii only (C) i and iii only (D) all are given

**2.** The sides of the triangle have length 9 , 13 , k where k is an integer . For how many values of k is the triangle obtuse? **[NSTSE 2009]**

(A) 11 (B) 12 (C)15 (D) 6

**3.** The value of 'x' in the given figure is : **[NSTSE 2009]**




(A) 15° (B) 20° (C) 25° (D) 30°

**4.** In a triangle PQR , $PQ^2 = QR^2 + PR^2$ , then the right angle is at : **[NSTSE 2010]**

(A) P (B) Q (C) R (D) any vertex

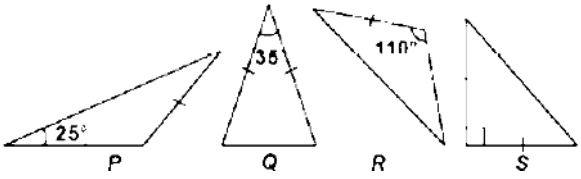
**5.** The degree measure of each of the three angles of a triangle is an integer. Which of the following could not be the ratio of their measures ? **[NSTSE-2010]**

(A) 2: 3 : 4 (B) 3 : 4 : 5 (C) 5 : 6 : 7 (D) 6: 7 : 8

**6.** A triangle has angle measurements of 32º, 43º and 105º. What kind of triangle is it? **[IMO-2010]**

(A) Equilateral (B) Isosceles (C) Scalene (D) None of these

**7.** Which of the following figures are obtuse isosceles triangle? **[IMO-2010]**

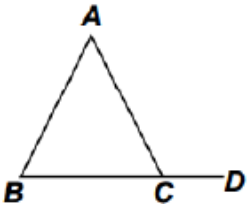



(A) P & R only (B) Q & R only (C) P, Q & R only (D) P, R & S only

**8.** A ladder that is 13 m long leans against a building. The bottom of the ladder is 5 m away from the base of the building. How far up the side of the building does the ladder reach? **[IMO-2010]**

(A) 18 m (B) 16 m (C) 12 m (D) 8 m

**9.** In the given figure, BC is produced to D and ∠BAC = 40º and ∠ABC = 70º. Find the value of ∠ACD. **[IMO-2011]**




(A) 130º (B) 140º (C) 170º (D) 110º

**10.** What type of angles are the two acute angles in a right triangle? **[NSTSE 2012]**

(A) Supplementary angles (B) Complementary angles

(C) Reflex angles (D) Obtuse angles

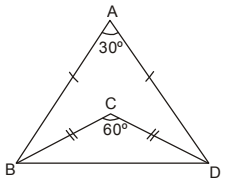
**11.** Of all the line segments that can be drawn to a given line from a given point outside it, which angle of inclination among the following alternative has the shortest length? **[NSTSE 2012]**

(A) 55º (B) 60º (C) 70º (D) 65º

**12.** The vertical angle of an isosceles triangle measure (5p – 18°) and one of the base angles measure 3p . What is the value of p ? **[NSTSE 2012]**

(A) 24 (B) 15 (C) 18 (D) 12

**13.** In the diagram, what is the measure of ∠ABC ? **[NSTSE 2012]**




(A) 45º (B) 30º (C) 15º (D) 65º

**14.** There are two buildings standing on the opposite sides of a park. A ladder, 17 m long, reaches a point on a building at a height of 15 m from the ground. The ladder is now turned to the other building on the other side of the park keeping its foot at the same point. The ladder now reaches a point on this building at a height of 8 m above the ground. Find the distance between the two buildings. **[IMO-2012]**

(A) 32 m (B) 18 m (C) 25 m (D) 23 m

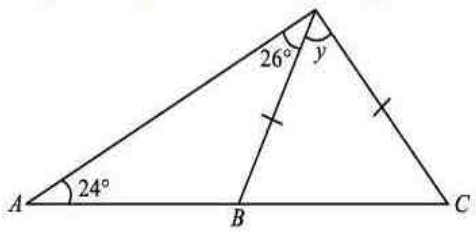
**15.** Which of the folowing are the angles of a right angled triangle ? **[NSTSE 2013]**

(A) 35 , 65 , 90 (B) 90 , 5 , 65 (C) 40 , 60 , 90 (D) 55 , 35 , 90

**16.** In a triangle ABC , if AB + BC = 10 cm , BC + CA = 12 cm , CA + AB = 16 cm , what is the perimeter ? **[NSTSE 2014]**

(A) 19 (B) 17 (C) 38 (D) 30

**17.** In the given figure ABC is a straight line. Find y. **[IMO-2013]**




(A) 50° (B) 80° (C) 100° (D) 130°

**18.** In two triangles PQR and LMN. PQ = QR, ∠ P = ∠ M and QR = LN.then which of the following is true? **[IMO-2014]**

(A) Triangles are congruent only
(B) Triangles are isosceles only
(C) Triangles are both congruent and isosceles
(D) None of these

**19.** Choose the correct option for ΔPQR. **[IMO-2014]**

(A) PQ – QR > PR (B) PQ + QR < PR
(C) PQ – QR <PR (D) PQ + PR < QR

**20.** The expression for the number of diagonals that we can make from one vertex of a n sided polygon is **[IMO-2014]**

(A) 2n + 1 (B) n – 2 (C) 5n + 2 (D) n – 3

# ANSWER KEY

## EXERCISE 01

### SECTION -A (FIXED RESPONSE TYPE)

### MULTIPLE CHOICE QUESTIONS

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | C | D | B | C | C | C | A | A | C | C | C | C | A | C | A |
| Ques. | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | | |
| Ans. | A | C | A | B | D | C | A | C | B | A | A | C | C | | |

### FILL IN THE BLANKS

**1.** Hexagon **2.** 1 **3.** Scalene **4.** 60°

**5.** Altitude **6.** Greater **7.** Greater **8.** Hypotenuse

### TRUE / FALSE

**1.** False **2.** True **3.** True **4.** True **5.** False

**6.** False **7.** True

### MATCH THE COLUMN

**1.** (A) → r, (B) → p, (C) → q, (D) → s, (E) → t, (F) → v, (G) → u

### SECTION -B (FREE RESPONSE TYPE)

### VERY SHORT ANSWER TYPE

**1.** 60° **2.** 40° **3.** 65°, 80° **4.** 53°, 127°. **5.** 130°

**6.** 144° **7.** No **8.** 26 m

### SHORT ANSWER TYPE

**9.** 55° **10.** **(i)** 70° **(ii)** 30° **(iii)** 42° **11.** x = 37°, y = 53°.

**12.** 60° **13.** 80°. **14.** Between 3cm and 27cm **15.** 13 m

## LONG ANSWER TYPE

**17.** 150° **18.** 6 sides **19.** 40°, 60°, 80°. **20.** 26°, 45°

**22.** 90° **23.** 18 m

# EXERCISE 02

## SECTION -A (COMPETITIVE EXAMINATION QUESTION)

## MULTIPLE CHOICE QUESTIONS

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | A | A | A | C | A | C | D | A | C | B | C | C |

## SECTION -B (TECHIE STUFF)

| Ques. | 13 | 14 |
|---|---|---|
| Ans. | A | B |

# EXERCISE 03

## PREVIOUS YEAR EXAMINATION QUESTIONS

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | C | A | C | C | D | C | A | C | D | B | C | C | C | D | D |
| Ques. | 16 | 17 | 18 | 19 | 20 | | | | | | | | | | |
| Ans. | A | B | D | C | D | | | | | | | | | | |