MATHEMATICS

Class-VII

Topic-06 CONGRUENCE OF TRIANGLES



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CONGRUENCE OF TRIANGLES

TERMINOLOGIES

Congruent figures, Congruent Triangles, SAS, ASA, AAS, SSS, RHS, Similar Figures, AAA Similarity, SSS Similarity, SAS Similarity.

INTRODUCTION

Have you seen the two stamps of same denomination, shaving blades of same company, sheets of the same letter pad,etc,... Now if you place one stamp over the other.what do you observe? One stamp covers the other completely and exactly.This means that the two stamps are of the same shape and size, such objects are said to be congruent. Two plane figures are congruent if each is a carbon copy of the other or if we put one

Two plane figures are congruent if each is a carbon copy of the other or if we put one object on other then they completely cover each other .This method of examining the congruence is called the method of superposition..

6.1 CONGRUENCE OF TRIANGLES

(a) Congruent figures



In the above figures {fig.(i) and fig.(ii)} both are equal in length, width and height, so these are congruent figures.

(b) Congruent triangles

Two triangles are **congruent** if and only if one of them can be made to superimpose on the other, so as to cover it exactly.



If two triangles $\triangle ABC$ and $\triangle DEF$ are congruent then there exist a one to one correspondence between their vertices and sides i.e. we get following six equalities

 $\angle A = \angle D, \ \angle B = \angle E, \ \angle C = \angle F$ and AB = DE, BC = EF, AC = DF.

If two $\triangle ABC \& \triangle DEF$ are congruent under $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$ one to one correspondence then we write $\triangle ABC \cong \triangle DEF$ we can not write as $\triangle ABC \cong \triangle DFE$ or $\triangle ABC \cong \triangle EDF$ or in other forms because $\triangle ABC \cong \triangle DEF$ have following one-one correspondence $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$.





Hence we can say that "two triangles are congruent if and only if there exists a one-one correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal.

(c) Sufficient conditions for congruence of two triangles

(i) SAS Congruence Criterion



Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

(ii) ASA Congruence Criterion:



Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

(iii) AAS Congruence Criterion:



If any two angles and a non included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.

(iv) SSS Congruence Criterion:



Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

(v) RHS Congruence Criterion:



Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.





(d) Congruence relation in the set of all triangles

By the definition of congruence of two triangles, we have following results.

(i) Every triangle is congruent to itself i.e. \triangle ABC $\cong \triangle$ ABC.

(ii) If $\triangle ABC \cong \triangle DEF$ then $\triangle DEF \cong \triangle ABC$.

(iii) If \triangle ABC $\cong \triangle$ DEF and \triangle DEF $\cong \triangle$ PQR then \triangle ABC $\cong \triangle$ PQR.

NOTE:

If two triangles are congruent then their corresponding sides and angles are also congruent by **cpct (corresponding parts of congruent triangles are also congruent).**

Illustration 6.1

Prove that : "Angles opposite to equal sides of an isosceles triangle are equal."

Sol. Given : $\triangle ABC$ in which AB = AC.

To Prove : $\angle B = \angle C$.

Construction : We draw the bisector AD of $\angle A$ which meets BC in D.



AB = AC[Given] \angle BAD = \angle CAD[AD is bisector of \angle A]And, AD = AD[Common side]By SAS criterion of congruence, we have \triangle ABD \cong \triangle ACD \Rightarrow \angle B = \angle C[By CPCT]Hence Proved.

Illustration 6.2

Prove that : "If two angles of a triangle are equal, then sides opposite to them are also equal."

Sol. Given : $\triangle ABC$ in which $\angle B = \angle C$. **To Prove :** AB = AC.

Construction : We draw the bisector of $\angle A$ which meets BC in D.

Proof : In $\triangle ABD$ and $\triangle ACD$ we have

 $\angle B = \angle C$ [Given] $\angle BAD = \angle CAD$ [AD is bisector of $\angle A$]AD = AD[Common side]By AAS criterion of congruence, we get $\triangle ABD \cong \triangle ACD$ AB = AC[By CPCT]Hence Proved.



 \Rightarrow



Illustration 6.3

In $\triangle ABC$, AB = 3 cm, BC = 4 cm and CA = 2 cm and in $\triangle DEF$, DE = 2 cm, EF = 3 cm and FD = 4 cm. Is the above given triangle are congruent. If they are congruent write out the pairs of equal angles.

Sol.



In triangles ABC and DEF, we have AB = EF = 3 cm, BC = FD = 4 cm and CA = DE = 2 cmSo, by SSS condition of congruence, we have $\triangle ABC \cong \triangle EFD$ Since, AB = EF, therefore angles opposite to these sides are equal. $\Rightarrow \qquad \angle C = \angle D$ Similarly, $BC = FD \Rightarrow \qquad \angle A = \angle E$ and $CA = DE \Rightarrow \qquad \angle B = \angle F$.

Illustration 6.4

Which of the following pairs of triangles are congruent?

- (i) △ABC, AB = 2 cm, AC = 4 cm, ∠A = 40°; △XYZ, XZ = 2 cm, YZ = 4 cm, ∠Z = 40°.
- (ii) △PQR, PQ = 5 cm, PR = 6 cm, ∠P = 55°; △DEF, DE = 6 cm, EF = 5 cm, ∠D = 55°.
- **Sol.** (i) In $\triangle ABC$ and $\triangle XYZ$, we have

AB = XZ = 2 cm, AC = YZ = 4 cm and $\angle A = \angle Z = 40^{\circ}$ Thus, in $\triangle ABC$ and $\triangle XYZ$, the two sides and the included angle of one triangle are equal to two sides and the corresponding included angle of the other.

So by SAS congruence condition, we have $\triangle ABC \cong \triangle ZXY$

(ii) In \triangle PQR, the included angle between PQ and PR is \angle P. In \triangle DEF, the included angle between DE and EF is \angle E. We have, PQ = EF = 5 cm and PR = DE = 6 cm but \angle P $\neq \angle$ E

So, the given triangles are not congruent.

Illustration 6.5

Show that the bisector of vertical angle of an isosceles triangle bisects the base at right angles.

Sol. Let $\triangle ABC$ be an isosceles triangle such that AB = AC. Let AD be the bisector of vertical angle $\angle A$ meeting BC in D.

Now, in Δs ABD and ACD, we have

AB = AC
 $\angle BAD = \angle CAD$ [Given]andAD = AD[\because AD is the bisector of $\angle A$]





1. In the given figure, O is the mid-point of AB and AC || BD. Show that $\triangle AOC \cong \triangle BOD$.







2. P is any point in the ∠ABC such that the perpendicular drawn from P on AB and BC are equal. Prove that BP bisects ∠ABC.

3. It is given that AB = EF, BC = DE, $AB \perp BD$ and $EF \perp CE$. Prove that $\triangle ABD \cong \triangle FEC$.



4. In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of \triangle ABC such that AX = AY. Prove that CX = BY.



5. Which Criteria can be used to prove congruence of 2 triangles ?





Two geometric figures having the same shape and size are known as **congruent figures**. Geometric figures having the same shape but different sizes are known as **similar figures**. Two triangles **ABC** and **DEF** are said to be similar if their

Corresponding angles are equal. i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ And,



- (ii) Corresponding sides are proportional.
 - i.e. $\frac{\Delta R}{DE} = \frac{RC}{DE} = \frac{\Delta C}{DE}$
- (a) Characteristic Properties of Similar Triangles :
- (i) (AAA Similarity) If two triangles are equiangular, then they are similar.
- (ii) (SSS Similarity) If the corresponding sides of two triangles are proportional, then they are similar.



(i)



(iii) (SAS Similarity) If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar. Lets understand this topic more clearly with the help of an example

Example : $\triangle ABC$ and $\triangle PQR$ are similar triangles such that $\angle A = 32^{\circ}$ and $\angle R = 65^{\circ}$, then $\angle B$ is :

R

Sol.

 $\Delta ABC \sim \Delta PQR$ $\angle A = \angle P, \ \angle B = \angle Q \text{ and } \angle C = \angle R$ $\therefore \ \angle P = 32^{\circ}, \ \angle C = 65^{\circ}$ In ΔABC $\Rightarrow \qquad \angle A + \angle B + \angle C = 180^{\circ}$ $\Rightarrow \qquad 32^{\circ} + \angle B + 65^{\circ} = 180^{\circ}$ $\Rightarrow \qquad \angle B = 180^{\circ} - 32^{\circ} - 65^{\circ}.$ $\Rightarrow \qquad \angle B = 180^{\circ} - 97^{\circ} = 83^{\circ}.$

Example : Two triangles ABC and PQR are similar, if BC : CA : AB = 3 : 6 : 5, find the value of $\frac{QR}{PR}$.

Sol. $\triangle ABC \sim \triangle PQR$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

B

	PQ	QR P
So	BC	AC
30,	QR -	PR
\rightarrow	QR	_ BC
	PR	AC
\rightarrow	QR	_ 3 _ 1
\rightarrow	PR	6 2





Concept Map







Summary.

- 1. Two figures are called **congruent** if they have same shape and same size. In other words, two figures are called congruent if they are having equal length, width and height.
- 2. Two triangles are **congruent** if and only if one of them can be made to superimposed on the other, so as to cover it exactly.
- 3. If two triangles △ABC and △DEF are congruent then there exist a one to one correspondence between their vertices and sides i.e. we get following six equalities ∠A = ∠D, ∠B = ∠E, ∠C = ∠F and AB = DE, BC = EF, AC = DF.
- **4.** Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle (SAS)
- **5.** Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.(ASA)
- **6.** If any two angles and a non included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.(AAS)
- **7.** Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.(SSS)
- 8. Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle. (R.H.S)
- **9.** Two congruent figures are equal in area but two figures having same area need not be congruent.
- **10.** Angles opposite to equal sides of an isosceles triangle are equal.
- **11.** If two angles of a triangle are equal, then sides opposite to them are also equal.
- **12.** The bisector of the vertical angle of an isosceles triangle bisects the base at right angles.











(A) 13 cm

8. If $\triangle ABC \cong \triangle DEF$, then the value of DF will be :



9. In the given figure, if AC \perp CD, BC \perp CD and AD = BD, than CA is equal to :



10. BC is the common base of \triangle ABC and \triangle DCB. Also AO = OD and BO = CO. Then which of the following statement is true by side -angle- side congruence property.







14. Which of the following are Criteria for congruence of triangles.

(A) RHS (B) SSS

(C) AAS

(D) All of these

FILL IN THE BLANKS

1. Two figures having same shape and size are known as _____

- 2. C.P.C.T means_____
- 3. If each angle of a triangle is equal, than measure of each angle is _____

4. Two circles are congruent if they have same _____

5. In a right angle triangle _____ is the longest side.

TRUE / FALSE

- **1.** Triangles with AAA condition are not congruent.
- 2. All squares are congruent .
- **3.** If two squares have equal areas, they are congruent.
- **4.** If two triangles are congruent then their corresponding sides and their corresponding angles are equal.
- **5.** If the hypotenuse of one right triangle is equal to the hypotenuse of another right triangle, then the triangle are congruent.

MATCH THE COLUMN

Colum	n – I	Colum	n – II
(A)	If $\triangle ABC \cong \triangle PQR$ then	(p)	SSA
(B)	In RHS, H stands for	(q)	SSS
(C)	Congruency condition does not hold true for	(r)	BC = QR
(D)	If 3 sides of one triangle are equal to corresponding 3 sides of other triangle, then Δ 's are congruent by	(s)	Hypotenuse
(E)	Sum of all angles of Δ	(t)	180°
(F)	Intersection point of medians	(u)	Orthocenter
(G)	Intersection points of altitudes	(v)	Centroid
(H)	Intersection points of angle bisectors	(w)	Incenter





SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

- 1. How many congruency criterias are there ?
- 2.

3.

In adjoining figure, PO = QO; $\angle P = \angle Q$

- (i) Is $\angle POR = \angle QOS$? Give reasons. (ii) Is $\triangle POR \cong \triangle QOS$? Give reasons.
- (iii) Is ∠PRO = ∠QSO ?
- In figure, AD = CD and AB = CB.



- (i) State the three pairs of equal parts in $\triangle ABD$ and $\triangle CBD$.
- (ii) Is $\triangle ABD \cong \triangle CBD$? Why or why not?
- (iii) Does BD bisect $\angle ABC$? Give reasons.
- 4. In quadrilateral PQRS, PQ = PS and PR is the bisector of $\angle P$. Show that $\triangle PQR \equiv \triangle PSR$. Is QR = SR ?
- 5.



In figure, can you use ASA congruence rule and conclude that $\triangle AOC \cong \triangle BOD$?

6. In figure AD is the bisector of \angle BAC, Prove that triangles ABD and ADC are congurent if AB = AC,







SHORT ANSWER TYPE

- **7.** Which of the following pairs of triangles are congruent? If they are congruent, write out the pairs of equal angles.
 - (i) $\triangle ABC: AB = 3 \text{ cm}, BC = 4 \text{ cm}, CA = 2 \text{ cm}$ $\triangle DEF: DE = 2 \text{ cm}, EF = 3 \text{ cm} \text{ and } FD = 4 \text{ cm}$
 - (ii) $\triangle PQR: PQ = 17 \text{ cm}, QR = 15 \text{ cm}, PR = 18 \text{ cm}$ $\triangle DEF: DE = 18 \text{ cm}, EF = 17 \text{ cm}, DF = 15 \text{ cm}.$
- 8. In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of $\triangle ABC$ such that AX = AY. Prove that CX = BY.



- **9.** Prove that measure of each angle of an equilateral triangle is 60°.
- **10.** In the given figure, AB = AC and OB = OC. Prove that $\angle ABO = \angle ACO$.



11. In figure AE || CD, AB = BC, AE = CD. If AC \perp CD & \angle AEB = 35°. Find \angle DBE.



12. In the given figure, ABC is an equilateral triangle; PQ || AC and AC is produced to R such that CR = BP. Prove that QR bisects PC.







LONG ANSWER TYPE

13. In figure, DA \perp AB, CB \perp AB and AC = BD. State the three pairs of equal parts in \triangle ABC and ΔDAB . Which of the following statements is meaningful ?



- $\triangle ABC \cong \triangle BAD$ (i)
- 14. If $\triangle ABC$ is an isosceles triangle such that AB = AC, then show that altitude AD from A on BC bisects BC.
- 15. In the given figure P is the mid point of BC. Also PQ = PR and, $PQ \perp AB$ and $PR \perp AC$. Then, prove that :



 $\Delta BPQ \cong \Delta CPR$ (i)

- AB = AC
- 16. In figure, it is given that AB = CD and AD = BC. Prove that $\triangle ADC \cong \triangle CBA$.



SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

- 1. In $\triangle ABC \angle A = 100$, AD bisects $\angle A$ and AD BC. Then $\angle B$ is equal to (A) 80 (B) 20 (C) 40 (D) 30
- 2. In the given figure it is given that AB = CF, EF = BD and $\angle AFE = \angle DBC$. Then congruence of $\triangle AFE$ and $\triangle CBD$ by



- (A) AAA congruence criterion.
- (B) SSS congruence criterion.
- (C) ASA congruence criterion.
- (D) SAS congruence criterion.





hypotenuse and the corresponding acute angle of another triangle, then the triangles are congruent.

(D) All of the above.

7. If AD, BE and CF, the altitude of \triangle ABC are equal. Then the \triangle ABC is :



(A) Equilateral triangle

(C) Isosceles triangle

(B) Right angle triangle (D) None of these

8. If $\triangle ABC$ is an isosceles triangle such that AB = AC and the altitude AD from A on BC, then : (B) BD = AC

(A) AB = DC

(C) BD = DC

(D) None of these

9. In the following which two triangles are congruent by side-side -side congruence property.







(A) 45°

10. A square board side 10 centimeters, standing vertically, is tilted to the left so that the bottom-right corner is raised 6 centimeters from the ground.



By what distance is the top-left corner lowered from its original position ? (A) 1 cm (B) 2 cm (C) 3 cm (D) 0.5 cm

11. In a right angled triangle ABC, P is mid point of AC. Which one is true ?

(A)
$$PA = \frac{AC}{2}$$
 (B) $PB = \frac{AC}{2}$ (C) $PA = PB$ (D) All of these

SECTION -B (TECHIE STUFF)

- **12.** In \triangle LMN, \angle L = 50° and \angle N = 60°. If \triangle LMN ~ \triangle PQR, then find \angle Q. (A)50° (B) 60° (C) 70° (D) none of these
- **13.** In figure if \triangle EDC ~ \triangle EBA, \angle EDC = 70° & \angle BEC = 115°. Find \angle EAB.



14. In the figure C is a right angle, $DE \perp AB$, AE = 6, EB = 7 and BC = 5. The area (in sq unit) of the quadrilateral EBCD is



(PREVIOUS YEAR EXAMINATION QUESTIONS)

1. For the congruence of $\triangle ABC$ and $\triangle PQR$, which one of the following sets of conditions is not sufficient? [NSTSE 2010]







2. Which of the following are corresponding part of the two given congruent figures?

[IMO-2010]













SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	В	С	D	С	D	D	С	С	D	А	В	D	С	D

FILL IN THE BLANKS

- 1.Congruent2.Corresponding parts of congruent triangles3.60°
- 4. Radius 5. Hypotenuse

ANSWER KEY

KHK:

TRUE / FALSE

1. True 2. False 3. True 4. True 5. False

MATCH THE COLUMN

 $\textbf{1.} \qquad (A) \rightarrow r, \, (B) \rightarrow s, \, (C) \rightarrow p, \, (D) \rightarrow q, \, (E) \rightarrow t, \, (F) \rightarrow v, \, (G) \rightarrow u, \, (H) \rightarrow w$

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

- **1.** 5
- 2. (i) vertically opposite angle (ii) Yes by ASA (iii) Yes by cpct
- 3. (i) AB = CB, AD = CD, BD = BD (ii) $\triangle ABD \cong \triangle CBD$ (iii) Yes
- **4**. Yes **5**. Yes

SHORT ANSWER TYPE

11. 70°





EXERCISE

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11
Ans.	С	D	В	D	А	D	А	С	В	В	D

SECTION -B (TECHIE STUFF)

Ques.	12	13	14	
Ans.	С	А	А	



(PREVIOUS YEAR EXAMINATION QUESTIONS)

Ques.	1	2	3	4	5	6	7	8	9	10	11
Ans.	С	С	С	С	А	В	С	В	С	В	Α

