MATHEMATICS

Class-VII

Topic-08

ALGEBRIC EXPRESSION

INDEX		
S. No.	**Topic**	**Page No.**
1.	**Theory**	**1 –12**
2.	**Exercise-1**	**13 – 17**
3.	**Exercise-2**	**17 – 19**
4.	**Exercise-3**	**20–21**
5.	**Answer Key**	**29 – 31**

CHAPTER 08

ALGEBRIC EXPRESSION

TERMINOLOGIES

Literals, Constant, Variable, Algebraic expressions, Monomial, Bionomial, Trinomial, Quadrinomial, Polynomials, Degree, Constant Polynomial, Algebraic identities, Linear polynomial, Quadratic polynomial, Cubic polynomial, Biquadratic polynomial, Factors, Like terms, Unlike terms.

INTRODUCTION

A combination of constants and variables connected by signs of fundamental operations (addition, subtraction, multiplication, division) is called algebraic expression .
Let's take an example : Parul has some apple. Ravina has 10 more than parul. Richa says that she has 3 more than number of apples of parul & Ravina together. So how do you get the number of apples that Richa has ?
After completion of this chapter you will be able to deal with such type of problems yourself.

8.1 ALGEBRAIC EXPRESSIONS

(a) Various Definitions and Concepts

Literals : The letters which are used to represent numbers are called literal numbers or **literals**. In 2xy, x & y are the literals.
Literal numbers obey all the rules (and signs) of addition, subtraction, multiplication and division of numbers along with the properties of these operations. $a \times b = ab$, $2 \times a = 2a$, $1 \times a = a$, $x \times 3 = 3x$ and $a \times a \times a \times$ .....$\times$ 15 times $= a^{15}$.
In a^5, 5 is called the **index or exponent** and a is called the **base**.

Constant : A term of the expression having no literal factor is called a **constant** term.
(i) In the binomial expression $5x + 7$, the constant term is 7. In short, a symbol having a fixed numerical value is called a constant.

(ii) In the trinomial expression $x^2 - y^2 - \frac{3}{2}$, the constant term is $-\frac{3}{2}$.

Variable : A symbol which takes various numerical values is called a **variable**.

Algebraic expression : A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an **algebraic expression**.

(b) Types of Algebraic Expressions

An algebraic expression is called a monomial, a binomial, a trinomial, a quadrinomial according as it contains one term, two terms, three terms and four terms respectively.

(i) Monomial : 5, $4x$, $7x^2y^3$, $-4xyz$, $\frac{2}{3}mn^2p^3$ are all monomials.

(ii) Binomial : $a + 5$, $4 - 3a$, $p^2 - 8qr$, $n^3 - 3$, $\frac{2}{3}x^2 + xyz^2$ are all binomials.

Note that $2 + 5$ is not a binomial, because $2 + 5 = 7$, which is a monomial.

(iii) Trinomial : $12x - 5y + 8$, $p^2 + q^2 + 4r^2$, $11 + abc + a^3$ are all trinomials.

NOTE:

That 3 + 6x + 6 is not a trinomial, because 3 + 6x + 6 = 6x + 9, which is a binomial.

(iv) Quadrinomials : $p^3 + q^4 + r^5 + 3pqr$, $a^2 - b^2 - c^2 - 7$, mn + np + pq + pqr are all quadrinomials.

(v) Polynomial : A **polynomial** is an algebraic expression with one or more terms.

For example : $6x, x^3 + 3x^2 + 9x + 7, 3x^2 - 4xy + 7y^2$ etc. are all polynomials.

(c) Degree of polynomial

The **degree** of a polynomial of one variable is the highest power of the variable in the given polynomial. For example $P(x) = 2x^3 + 3x^2 - 6x + 4$. The highest power of x in all terms of polynomial is 3. Hence, the degree of the polynomial is 3.

We can classify polynomial according to its degree.

(i) Constant polynomial : Polynomial having degree zero is known as **constant polynomial**.

For ex : 7, 8, $\frac{3}{2}$.

(ii) Linear polynomial : Polynomial having degree one is known as **linear polynomial**.

For ex : 2x – 5, x + 3.

(iii) Quadratic polynomial : Polynomial having degree two is known as **quadratic polynomial**.

For ex : $x^2 + 1, 7x^2, x^2 + 2x - 1$.

(iv) Cubic polynomial : Polynomial having degree three is known as **cubic polynomial**.

For ex : $7x^3 + 5x^2 + 1, x^3 - x + 1$.

(v) Biquadratic polynomial : Polynomial having degree four is known as **biquadratic polynomial**.

For ex : $x^4 + 1, x^4 + x^2 + 1$.

(d) Factors

Each term in an algebraic expression is a product of one or more number(s) and/or literal number(s). These number(s) and/or literal number(s) are known as the factors of that term.

(i) In the binomial 8ab + 3c, 8ab and 3c are two terms. In the term 8ab, 8, a and b are its factors. Clearly, number 8 is the numerical factor, and a and b are literal factors.

(ii) In the binomial expression – ab – 5, the term – ab has – 1 as the numerical factor while a and b are literal factors.

(e) Coefficient

In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the product of the factors.

Consider the term – 5ab in the binomial – 5ab + 7. The coefficient of a in the term – 5ab is – 5b, the coefficient of b is – 5a and the coefficient of ab is – 5.

(f) Like terms

The terms having the same literal factors are called like or similar terms.

In the algebraic expression $12a^2 - 15b^2 + b^2 - 17a^2 + 8ab + 9$, we have, $12a^2$ and $-17a^2$ as like terms and also $-15b^2$ and b^2 are like terms.

(g) Unlike terms

The terms not having same literal factors are called unlike or dissimilar terms.
In the algebraic expression $3p^2q + 5pq^2 - 7pq - 9qp^2$, $5pq^2$ and $-7pq$ are unlike terms.

Ask yourself

1. Write the cofficient of x^2 in the following :
(i) $-3x^2$ **(ii)** $2x^2yz$

2. Identify the like term in $a^2 + b^2 - 2a^2 + c^2 + 4a$

3. Write all the terms of the following expression
(i) 2x+3y–5z **(ii)** 3xyz+5m

4. Identify the monomial, binomial,trinomial and quadrinomials from the following expressions:
(i) 7 **(ii)** 3x+5y+7z

5. Determine the degree of the polynomial $x^2 + 5x + 7$

Answers

1. **(i)** – 3 **(ii)** 2yz **2.** a^2 and $-2a^2$

3. **(i)** 2x, 3y, 5z **(ii)** 3xyz, 5m **4.** **(i)** monomial **(ii)** trinomial

5. 2

8.2 OPERATION ON ALGEBRAIC EXPRESIONS

(a) Addition or subtraction of like terms

The sum or difference of several like terms is another like term whose coefficient is the sum or difference of those like terms.

Illustration 8.1

Add the following : 3xy, 10xy and 5xy.

Sol. The sum of the numerical coefficients of the given like terms is 3 + 10 + 5 = 18.
Thus, the sum of the given like terms is another like term whose numerical coefficient is 18.
Hence, 3xy + 10xy + 5xy = 18xy.

Illustration 8.2

Add the following : $-2p^2q$, $-9p^2q$, $-14p^2q$ and $-5p^2q$.

Sol. The sum of the numerical coefficients (without negative sign) is : 2 + 9 + 14 + 5 = 30
Hence, $-2p^2q - 9p^2q - 14p^2q - 5p^2q = -30p^2q$.
In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.

Illustration 8.3

Add the following : 3x + y + 4 and 4x + 3y + 7.

Sol. Horizontal Method

(3x + y + 4) + (4x + 3y + 7)

= (3x + 4x) + (y + 3y) + (4 + 7)

= (3 + 4)x + (1 + 3)y + (4 + 7)

= 7x + 4y + 11

OR

Column Method

$$\begin{array}{r} 3x + y + 4 \\ +\ 4x + 3y + 7 \\ \hline 7x + 4y + 11 \end{array}$$

Illustration 8.4

Add the following : 3x + 4y + 5z and 2x – 3y – 4z.

Sol. Horizontal Method

(3x + 4y + 5z) + (2x – 3y – 4z)

= (3x + 2x) + (4y – 3y) + (5z – 4z)

= (3 + 2)x + (4 – 3)y + (5 – 4)z

= 5x + y + z

OR

Column Method

$$\begin{array}{r} 3x + 4y + 5z \\ +\ 2x - 3y - 4z \\ \hline 5x + y + z \end{array}$$

NOTE:

To subtract an expression from another, we change the sign (from '+' to '–' and from '–' to '+') of each term of the expression to be subtracted and then add the two expressions.

Illustration 8.5

Subtract :

(i) 3p from 7p **(ii)** – 8x from 9x

(iii) – 3a from 7a **(iv)** – 9b from – 2b

Sol. **(i)** 7p – 3p = (7 – 3)p = 4p

(ii) 9x – (–8x) = 9x + 8x =(9 + 8) x = 17x

(iii) 7a – (–3a) = 7a + 3a = (7 + 3)a = 10a

(iv) – 2b – (– 9b) = – 2b + 9b = (– 2 + 9)b = 7b

Illustration 8.6

What should be subtracted from $2p^3 – 4p^2 + 5p – 6$ to obtain $p^2 – 2p + 1$?

Sol. Let X denote the required expression.

Then, $(2p^3 – 4p^2 + 5p – 6) – X = p^2 – 2p + 1$

Hence, required expression

$X = (2p^3 – 4p^2 + 5p – 6) – (p^2 – 2p + 1)$

$X = 2p^3 – 4p^2 + 5p – 6 – p^2 + 2p – 1$

$X = 2p^3 – 4p^2 – p^2 + 5p + 2p – 7$

$X = 2p^3 – 5p^2 + 7p – 7$

NOTE:

When a grouping symbol preceded by, '–' sign is removed or inserted, then the sign of each term of the corresponding expression is changed (from '+' to '–' and from '–' to '+').

Illustration 8.7

Simplify : $2x - \{4y - (3x - 5y)\}$.

Sol. We first remove the innermost grouping symbol ( ) and then braces { }.

Thus, we have

$2x - \{4y - (3x - 5y)\}$

$= 2x - \{4y - 3x + 5y\}$ [Removing ( )]

$= 2x - \{9y - 3x\}$

$= 2x - 9y + 3x$

$= 2x + 3x - 9y = 5x - 9y.$

Illustration 8.8

Simplify and find the value of the following expression when a = 2 and b = 3 :

$4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{- b^3 + 4(a - 3)\}]$

Sol. Proceeding outward from the innermost bracket,

$4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{- b^3 + 4(a - 3)\}]$

$= 4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{- b^3 + 4a - 12\}]$

$= 4a^2 + 4b^2 + 8ab - [4a^2 + 4b^2 - 8ab + b^3 - 4a + 12]$

$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 + 8ab - b^3 + 4a - 12$

$= 4a^2 - 4a^2 + 4b^2 - 4b^2 + 8ab + 8ab - b^3 + 4a - 12$

$= (4 - 4)a^2 + (4 - 4)b^2 + (8 + 8)ab - b^3 + 4a - 12$

$= 16ab - b^3 + 4a - 12$

Thus value of this expression for a = 2 and b = 3 is :

$16 \times 2 \times 3 - (3)^3 + 4 \times 2 - 12$

$= 96 - 27 + 8 - 12 = 65.$

(b) Multiplication of algebraic expressions

Following rules of signs and the laws of exponents used in multiplication.

1. Rules of signs in multiplication :

The product of two factors with like signs is positive, and the product of two factors with unlike signs is negative. Thus if a and b are two positive numbers then

(i) $(+ a) \times (+ b) = + (ab)$; eg $2 \times 3 = 6$

(ii) $(+a) \times (- b) = - (ab)$; eg $\frac{2}{3} \times \frac{-5}{7} = \frac{-10}{21}$

(iii) $(- a) \times (+ b) = - (ab)$; eg $\frac{-2}{3} \times \frac{5}{7} = \frac{-10}{21}$

(iv) $(- a) \times (- b) = + (ab)$; eg $\frac{-2}{3} \times \frac{-5}{7} = \frac{10}{21}$

2. Laws of exponents in multiplication

Law of exponents in multiplication is given as :

$a^m \times a^n = a^{m+n}$; e.g. $x^5 \times x^2 = x^{5+2} = x^7$

(i) Multiplication of a monomial by a monomial

Illustration 8.9

Multiply : **(i)** $2x^3y^2 \times 5\,x^2y$ **(ii)** $\frac{4}{5}ab \times \frac{-3}{2}a^2bc$ **(iii)** $\frac{-2}{3}x\ \frac{3}{4} \times y \times -\frac{4}{5}z$

Sol. **(i)** $2x^3y^2 \times 5\,x^2y = (2 \times 5)\,x^{3+2}\,y^{2+1} = 10x^5y^3$

(ii) $\frac{4}{5}ab \times \frac{-3}{2}a^2bc = \left(\frac{4}{5} \times \frac{-3}{2}\right)a^{1+2}\,b^{1+1}c^1 = \frac{-6}{5}a^3b^2c$

(iii) $\frac{-2}{3}x \times \frac{3}{4}y \times -\frac{4}{5}z = \left(\frac{-2}{3} \times \frac{3}{4} \times \frac{-4}{5}\right)x \times y \times z = \frac{2}{5}xyz$

Thus in multiplication of algebraic expression :
(i) Write the product of the numerical coefficients.
(ii) Write all the different letters occurring in the algebraic expressions giving to each letter an exponent (power) equal to the sum of all the exponents of that letter in the given expressions.
(iii) The sign of the product is minus if there is an odd number of negative factors and plus if there is an even number of negative factors.

(ii) Multiplication of a Binomial by a monomial

In order to multiply a binomial by a monomial use the following rule :
$a \times (b + c) = a \times b + a \times c$.

Illustration 8.10

Multiply : **(i)** $3x^2 + 4xy$ by $2x$ **(ii)** $-4a\,[a + 3b]$ **(iii)** $-4a\,[a - 3b]$

Sol. **(i)** $2x\,[3x^2 + 4xy] = 2x \times 3x^2 + 2x \times 4xy = 6x^3 + 8x^2y$

(ii) $-4a\,[a + 3b] = -4a \times a - 4a \times 3b = -4a^2 - 12ab$

(iii) $-4a\,[a - 3b] = -4a \times a - 4a \times (-3b) = -4a^2 + 12\,ab$.

(iii) Multiplication of a Binomial by a binomial

In multiplication of a binomial by binomial we will use the law of multiplication of a binomial by a monomial twice.

Illustration 8.11

(i) $(a + b)(c + d)$ **(ii)** $(2x^2 + 3y)(3x^2 - 2y)$

Sol. **(i)** $(a + b)(c + d) = a(c + d) + b(c + d)$
$= a \times c + a \times d + b \times c + b \times d$
$= ac + ad + bc + bd$

(ii) $(2x^2 + 3y)(3x^2 - 2y)$
$= 2x^2(3x^2 - 2y) + 3y(3x^2 - 2y)$
$= 2x^2 \times 3x^2 + 2x^2 \times (-2y) + 3y \times 3x^2 + 3y \times (-2y)$
$= 6x^4 - 4x^2y + 9x^2y - 6y^2$

(c) Division of algebraic expressions

Division is the inverse process of multiplication.
When we divide one expression by another, we find a third expression which when multiplied by the second gives the first , i.e., if $a \div b = x$ then $a = bx$. In $a \div b = x$, **'a'** is called the **Dividend**, **'b'** the **Divisor** and **'x'** is called the **Quotient**.

Rules of Signs in Division :

(i) When the dividend and the divisor have the same signs, the quotient has the plus sign.
(ii) When the dividend and the divisor have opposite signs, the quotient has the negative sign.

(i) Division of a monomial by another monomial

Illustration 8.12

Divide : **(i)** 15abc by 5b **(ii)** $36a^3b^5c^6$ by $-12a^2bc$

(iii) $-156x^3y^5z^8$ by $-13x^2y^2z^3$

Sol. **(i)** Quotient $= \frac{15abc}{5b} = \frac{3 \times 5 \times a \times b \times c}{5 \times b} = 3ac$

(ii) Quotient $= \frac{36a^3b^5c^6}{-12a^2bc} = \frac{36}{-12} \times \frac{a^3}{a^2} \times \frac{b^5}{b} \times \frac{c^6}{c} = -3ab^4c^5$

(iii) Quotient $= \frac{-156x^3y^5z^8}{-13x^2y^2z^3} = \frac{-156}{-13} \times \frac{x^3}{x^2} \times \frac{y^5}{y^2} \times \frac{z^8}{z^3} = 12xy^3z^5$

(ii) Division of a polynomial by another monomial

Divide each term of the polynomial by the monomial and then write the resulting quotients.

Illustration 8.13

Divide : **(i)** $-4x^3 - 6x^2 + 8x$ by $2x$ **(ii)** $3x^4y - 4x^3y^2 + 5x^2y^3$ by $-6x^2y$

(iii) $-a^8b^5 + \sqrt{3}\, a^7b^6 - a^6b^7$ by $-2a^8b^6$

Sol. **(i)** Quotient $= \frac{-4x^3 - 6x^2 + 8x}{2x} = \frac{-4x^3}{2x} - \frac{6x^2}{2x} + \frac{8x}{2x} = -2x^2 - 3x + 4.$

(ii) Quotient $= \frac{3x^4y - 4x^3y^2 + 5x^2y^3}{-6x^2y} = \frac{3x^4y}{-6x^2y} - \frac{4x^3y^2}{-6x^2y} + \frac{5x^2y^3}{-6x^2y} = -\frac{x^2}{2} + \frac{2xy}{3} - \frac{5y^2}{6}$.

(iii) Quotient $= \frac{-a^8b^5 + \sqrt{3}a^7b^6 - \frac{1}{6}a^6b^7}{-2a^8b^6} = \frac{-a^8b^5}{-2a^8b^6} + \frac{\sqrt{3}a^7b^6}{-2a^8b^6} - \frac{\frac{1}{6}a^6b^7}{-2a^8b^6} = \frac{1}{2b} - \frac{\sqrt{3}}{2a} + \frac{b}{12a^2}$

Ask yourself

1. Find the value of expression $z^3 - 2(z - 10)$ for $z = 10$

2. Find the product of $4a^2$, $-6b^2$ and $3a^2b^2$

3. $P = 3x - 4y - 8z$, $Q = -10y + 7x + 11z$
$R = 19z - 6y + 4x$, find $P - Q + R$

4. Solve $(x + 4)(x + 3) - (x - 4)(x - 3)$

5. Divide $7p^2qr^5$ by $343p^5q^4r^{-3}$

Answers

1. 1000 **2.** $-72\, a^4\, b^4$ **3.** 0 **4.** 14x

5. $\frac{r^8}{49p^3q^3}$

8.3 ALGEBRAIC IDENTITIES

An identity is an equality, which is true for all values of the variables.
The following three identities are very important.

Identity 1 : $(a + b)^2 = a^2 + 2ab + b^2$
Proof : we have :

$(a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$ [Since ba = ab]

$\therefore (a + b)^2 = a^2 + 2ab + b^2$.

Identity 2 : $(a – b)^2 = a^2 – 2ab + b^2$.
Proof : we have

$(a – b)^2 = (a – b)(a – b)$
$= a(a – b) – b(a – b)$
$= a^2 – ab – ba + b^2$
$= a^2 – ab – ab + b^2$ [Since ba = ab]
$= a^2 – 2ab + b^2$.

$\therefore (a – b) = (a^2 – 2ab + b^2)$

Identity 3 : $(a + b)(a – b) = a^2 – b^2$
Proof We have :

$(a + b)(a – b) = a(a – b) + b(a – b)$
$= a^2 – ab + ba – b^2$
$= a^2 – b^2$ [Since ba = ab]

$\therefore (a + b)(a–b) = a^2 – b^2$.

(a) Applications of the above identities

Illustration 8.14

Find each of the following products :

(i) $(3x +2y)(3x +2y)$ **(ii)** $(4x^2 + 5)(4x^2 + 5)$

(iii) $(2x – 5y)^2$ **(iv)** $(3x^2 +2y^2)(3x^2 – 2y^2)$

Sol. **(i)** $(3x +2y)(3x +2y) = (3x +2y)^2$
$= (3x)^2 + (2y)^2 + 2(3x)(2y)$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]
$= 9x^2 +4y^2 + 12xy$.
$\therefore (3x + 2y)(3x + 2y) = 9x^2 + 4y^2 + 12xy$.

(ii) $(4x^2 + 5)(4x^2 + 5) = (4x^2 + 5)^2$
$= (4x^2)^2 + 5^2 + 2(4x^2)5$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]
$= 16x^4 +25 + 40x^2$.

(iiii) $(2x – 5y)^2 = (2x)^2 + (5y)^2 – 2(2x)(5y)$ [Using $(a – b)^2 = a^2 + b^2 – 2ab$]
$= 4x^2 + 25y^2 – 20xy$.

(iv) $(3x^2 + 2y^2)(3x^2 – 2y^2) = (3x^2)^2 – (2y^2)^2$ [$\therefore (a + b)(a – b) = (a^2 – b^2)$]
$= (9x^4 – 4y^4)$.

Illustration 8.15

If $x + \frac{1}{x} = 5$, find the values of :

(i) $x^2 + \frac{1}{x^2}$ **(ii)** $x^4 + \frac{1}{x^4}$

Sol. **(i)** $x + \frac{1}{x} = 5 \Rightarrow \left(x + \frac{1}{x}\right)^2 = (5)^2$ [on squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 25$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = (25 - 2)$

$\Rightarrow x^2 + \frac{1}{x^2} = 23$

(ii) $x^2 + \frac{1}{x^2} = 23$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$ [on squaring both sides]

$\Rightarrow x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right) = 529$

$\Rightarrow x^4 \times \frac{1}{x^4} + 2 = 529 \Rightarrow x^4 + \frac{1}{x^4} = (529 - 2)$

$\Rightarrow x^4 + \frac{1}{x^4} = 527.$

Ask yourself

1. If $x + \frac{1}{x} = 8$, find the values of :

(i) $x^2 + \frac{1}{x^2}$ **(ii)** $x^4 + \frac{1}{x^4}$

2. Evaluate : $(99)^2$

3. Find the product : $(x - 1)(x + 1)(x^2 + 1)$

4. If $x = 152$, $y = -91$ find the value of $9x^2 + 30xy + 25y^2$.

5. Evaluate:

(i) $(2x - 3)^2$ **(ii)** $\left(5x^2 + 7xy\right)^2$

Answers

1. **(i)** 62 **(ii)** 3842 **2.** 9801 **3.** $x^4 - 1$

4. 1 **5.** **(i)** $4x^2 - 12x + 9$ **(ii)** $25x^4 + 70x^3y + 49x^2y^2$

Add your knowledge

1. Can you divide $x^2 + 5x + 6$ by $x + 3$ and tell quotient and remainder ?
Answer to your query is as explained below:

$$
\begin{array}{r|l}
x+3 & x^2 + 5x + 6 \quad (x+2 \\
 & x^2 + 3x \\
 & - \quad - \\
\hline
 & 2x + 6 \\
 & 2x + 6 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Quotient = $x + 2$
Remainder = 0

EXPLANATION :

(i) Divide the first term (x^2) of the dividend by the first term (x) of the divisor.

The result $x^2 \div x = x$ is the first term of the quotient.

(ii) Multiply the divisor $x + 3$ by x, the first term of the quotient.

(iii) Subtract the product $(x + 3)\, x = x^2 + 3x$ from the dividend $x^2 + 5x + 6$. i.e. $(x^2 + 5x + 6) - (x^2 + 3x) = 2x + 6$

(iv) Proceed with this remainder $2x + 6$ as with the original dividend i.e., divide $2x$ by x, The result
$2x \div x = 2$ is the second term of the quotient.

(v) Multiply the divisor $(x + 3)$ by 2, the second term of the quotient. Now subtract $2(x + 3)$ from $2x + 6$
i.e., $2x + 6 - 2(x + 3) = 2x + 6 - 2x - 6 = 0$. The remainder is 0.

Hence the required quotient = $x + 2$.

2. Some important identities related to cubes

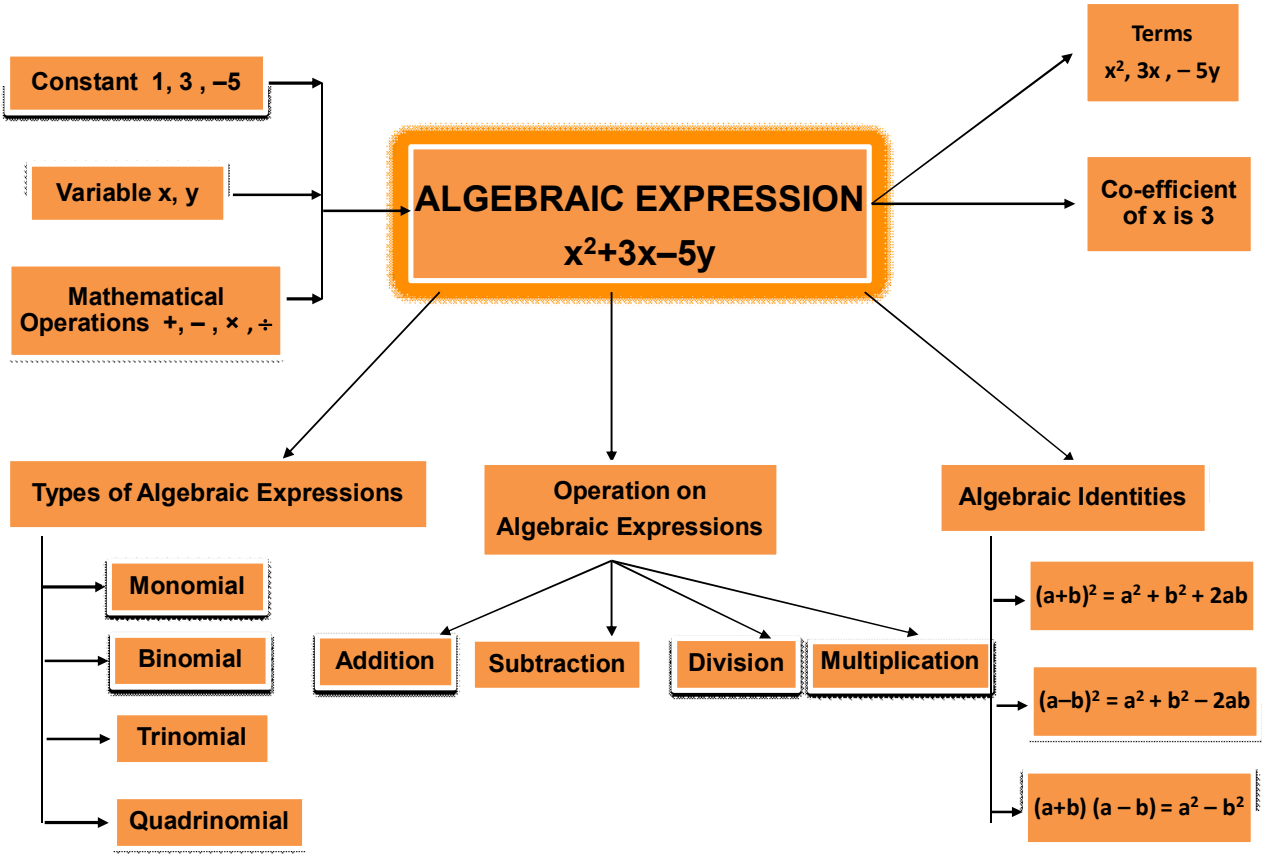
(a) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(b) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(c) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(d) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Concept Map


Constant 1, 3 , –5
Variable x, y
Mathematical Operations +, – , × , ÷
ALGEBRAIC EXPRESSION
x²+3x–5y
Terms
x², 3x , – 5y
Co-efficient of x is 3
Types of Algebraic Expressions
Monomial
Binomial
Trinomial
Quadrinomial
Operation on Algebraic Expressions
Addition
Subtraction
Division
Multiplication
Algebraic Identities
(a+b)² = a² + b² + 2ab
(a–b)² = a² + b² – 2ab
(a+b) (a – b) = a² – b²

Summary

1. The letters which are used to represent numbers are called literal numbers or **literals**. In 2xy, x & y are the literals.

2. A symbol which takes various numerical values is called a **variable**.

3. A term of the expression having no literal factor is called a **constant** term.

4. Coefficient is the numerical factor in a term . Sometimes , any factor in a term is called the coefficient of the remaining part of the term.

5. Any combination of letter s or of numerals and letters connected by the symbol +,– × ÷, is called an algebraic expression.

6. An expression is called monomial, binomial, trinomial, multinomial (polynomial) according as it has one,two,three,several terms.

7. A **polynomial** is an algebraic expression with one or more terms.

8. The **degree** of a polynomial of one variable is the highest power of the variable in the given polynomial.

9. Polynomial having degree zero,one,two, three , four is known as **constant, linear, quadratic, cubic, biquadratic polynomial respectively.**.

10. Each term in an algebraic expression is a product of one or more number(s) and/or literal number(s). These number(s) and/or literal number(s) are known as the factors of that term.

11. The terms having same literals factors are called like terms otherwise they are called unlike terms.

12. The sum of several like terms is another like term whose coefficient is the sum of the coefiicient of the like terms.

13. To subtrract one expression from another, we change the sign of each term of the expression to be subtracted and then add it to the expression from which subtraction was to be made.

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

1. The algebraic expression of the statement 'product of numbers a and b subtracted from 7' is

(A) ab – 7 (B) 7 – ab (C) ab (D) 7ab

2. $2x^2 + 3x + 7$ is

(A) monomial (B) binomial (C) trinomial (D) quadrinomial

3. Degree of 7

(A) 0 (B) 1 (C) 2 (D) 3

4. x^3 is

(A) constant polynomial (B) linear polynomial

(C) quadratic polynomial (D) cubic polynomial

5. Numerical coefficient of x in –7xyz

(A) 7 (B) –7 (C) –7yz (D) 7yz

6. Simplify : $(a^2 + b^2 + 2ab) + (a^2 + b^2 – 2ab)$.

(A) $–2a^2 + 2b^2$ (B) $2a^2 – 2b^2$ (C) $2a^2 + 2b^2$ (D) $(2a^2 – 2b^2)$

7. Simplify : $2x – [3y– \{2x – (y – x)\}]$.

(A) 5x + 4y (B) 5x – 4y (C) – 5x – 4y (D) – 5x + 4y

8. $– m – [m + \{m + n – 2m – (m – 2n)\} – n] =$

(A) 2n (B) 3n (C) – 2n (D) – 3n

9. Subtract $2x^3 + x^2 – 4x – 1$ from $5x^3 + 5x^2 + 9$.

(A) $–3x^3 + 4x^2 + 4x +10$ (B) $3x^3 + 4x^2 + 4x +10$

(C) $3x^3 + 4x^2 + 4x –10$ (D) $– (3x^3 + 4x^2 + 4x +10)$

10. $a^4 + 4a^2b^2 + b^4$ is more than $a^4 – 8a^2b^2 + b^4$ by :

(A) $12a^2b^2$ (B) $–12a^2b^2$ (C) $2a^4 + 2b^4$ (D) None of these

11. The algebraic expression $\left(\frac{1}{4}x^2y^2z^2\right) \times 3x \times \left(\frac{3}{2}y^2z\right)$ when expressed as monomial is :

(A) $\frac{9}{8}x^3y^4z^3$ (B) $\frac{1}{2}x^2y^2z^2$ (C) $\frac{8}{9}x^3y^4z^3$ (D) $\frac{3}{4}x^3y^2z$

12. Value of $x^2 – xy + y^2$ when x = 0 & y =1 is :

(A) –1 (B) 0 (C) 1 (D) 4

13. If x = 1, y = – 1 and z = – 1, then the value of $\frac{x^2yz^2}{3}$ is :

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) 1 (D) – 1

14. The value of $\frac{97x97-87x87}{10}$ is :

(A) 10 (B) 97 (C) 87 (D) 184

15. If $(a + b)^2 = a^2 + b^2 + 2ab$ and ab = 6 ; a + b = 5, find value of $a^2 + b^2$.

(A) 11 (B) 12 (C) 13 (D) 16

16. The mathematical expressionwhen $\frac{96\times96-4\times4}{96-4}$ simplified using the identity $a^2 – b^2 = (a + b) (a – b)$ results in the value :

(A) 100 (B) 105 (C) 82 (D) 94

17. Square of $3x^2 – 6y^2$ is :

(A) $9x^4 – 36y^4$ (B) $9x^4 + 36y^4 – 36x^2y^2$

(C) $9x^4 – 36y^4 + 36x^2y^2$ (D) $9x^4 – 36y^4 – 36x^2y^2$

18. $(4x^2 + 3y^2) (4x^2 – 3y^2)$ is :

(A)$16x^4 + 9y^4 – 24x^2y^2$ (B)$16x^4 + 9y^4 + 24x^2y^2$

(C)$16x^4 + 9y^4$ (D) $16x^4 – 9y^4$

19. Find the continued product : $(x + 2) (x – 2) (x^2 + 4)$

(A) $x^4 – 8$ (B) $x^2 – 16$ (C) $x^4 – 16$ (D)$x^4 – 4$

FILL IN THE BLANKS

1. Expression having 3 terms is known as _____

2. A symbol which takes various numerical values is called a _____

3. Degree of a constant term is _____

4. Numerical Coefficient of z in –7xyz is _____

5. Dividend = _____ × quotient + remainder

6. (2x+3y) + (5x–7y)= _____

7. (x + 2) (x – 2) =_____

8. $(a + 3)^2$=_____

TRUE / FALSE

1. Polynomial having degree four is known as cubic polynomial.

2. 5x + 2x is a monomial.

3. Highest power of a variable is known as Degree of polynomial.

4. 4b , –4b are like terms.

5. If x – y = 4 then y = x + 3

6. $\frac{8x^3yz^5}{4xyz} = 2x^2z^4$

7. $(2a+3b)^2 = 4a^2 + 9b^2 + 6ab$

8. $(2a+3b)(2a-3b) = 2a^2 - 3b^2$

MATCH THE COLUMN

1.

	Column – I		**Column – II**
(A)	Sum of x and y	(p)	3x – y
(B)	Product of x and y added to 3.	(q)	xy + 3
(C)	Thrice the difference of x and y	(r)	3 (x – y)
(D)	Twice a number y added to x	(s)	x + y
(E)	y subtracted from 3 times x.	(t)	2y + x
(F)	$(a - b)^2$	(u)	$(a^2 + 2ab + b^2)$
(G)	$(a + b)^2$	(v)	(a – b) (a + b)
(H)	$a^2 - b^2$.	(w)	$(a^2 - 2ab + b^2)$
(I)	Degree of linear polynomial	(x)	one

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. Name the polynomial according to no. of terms and according to degree.

(i) x + 7 **(ii)** $2x^3 + 3x^2 + 7$ **(iii)** 2x + x + 3

(iv) 7 + 3 + 2 **(v)** 2x + 4x – 2x

2. Write the numerical coefficient of each term in the following :

(i) $12x^2y + 7xy - xy^2$ **(ii)** $5A^2 + 7AB + 8$

(iii) $-A^3 + B^2 + 8AB + 7$ **(iv)** x + 9

3. Write the constant term of algebraic expresssion $3x^2 + 5x - 7$

4. Write all the terms of algebraic expression $2x^2 + 3y - 5x + 4$

5. Add : 4ab, – 7ab, – 10ab

6. Add 7x + 4 and 3x – 5

7. Subtract 7xyz from – 3xyz

8. Solve : $3xy^2 \times 5xyz$.

9. Solve : $2x(x^2 + y)$.

10. Solve : $-23x(5x + y)$.

11. Divide : $-8x^3$ by $-2x$.

12. Divide : $63x^2y^3$ by $-7xy^2$.

13. Divide : $x^5 + 4x^4 - 3x^2$ by x^2.

14. Divide : $8x^4 - 32x^3 + 16x^2$ by $-4x^2$.

15. Solve : $(x - y)(x + y)$.

16. The value of is $(47)^2 + (43)^2 - 2(47)(43)$:

17. The value of $\frac{7.98 \times 7.98 - 2.02 \times 2.02}{5.96}$ will be :

SHORT ANSWER TYPE

18. Write the following in to algebraic language.
(i) Difference of P and Q (P > Q).
(ii) The product of P and Q is subtracted from sum of P and Q.
(iii) Total cost of x books at Rs 20 per book and y pens at Rs.10 per pen.
(iv) When x is divided by y the quotient is z.
(v) Three fourth of x is added to two fifth of y gives 18

19. Write the statement for the following algebraic language :
(i) $x + y + z$ **(ii)** $(x + y)(x - y)$ **(iii)** $2x + 3y$
(iv) $\frac{2x}{5y} = 7$ **(v)** $\frac{2}{3}x + \frac{1}{5}y + \frac{7}{2}z$

20. Identify the like term in each of the following expression
(i) $xy^2 + 3xy^2 - 4x^2y^2 - 5xy^2 + 7x^2y$ **(ii)** $2x^3 - 3x^2 + 5x + 7x^2$

21. Find the sum of f(x) & g(x) where, $f(x) = 4x^5 + 3x^3 + 4x^2 + x + 1$ and $g(x) = 5x^4 + x^5 + x^3 + 3$.

22. Subtract g(x) from f(x), where $f(x) = 2 + x^2 + 4x^3$ and $g(x) = x^4 + x^2 + 3x + 5$.

23. How much is $3p^4 + p^3 - 2p^2 + p + 4$ greater than $2p^3 + 7p^2 - 5p + 6$?

24. If A = 2, B = – 1, C = – 3, find the value of :
(i) A + B – C **(ii)** (A + B) (B + C) (C + A) **(iii)** (A – B) (B – C) (C – A)
(iv) $A^3 + B^3 + C^3$ **(v)** $A^2 + B^2 + 2AB$ **(vi)** (A + B) (A – B)
(vii) – 2A + 3B – 4C

25. Add $3x^2 - 4x + 1$; $-4x^2 + 5x + 5$ and $2x^2 - 3x - 2$.

26. Divide $3a^2b^3c^5 + 7a^7b^3c^2 - 5a^4b^2c^6$ by a^2bc^2

27. Find value of $\frac{x^2}{3} - \frac{y^2}{4} + \frac{z^2}{5}$, when x = 3, y = 4, z = 5

28. Evaluate :
(i) $(3x - 7y)^2$ **(ii)** $(4a + 3bc)^2$

29. Factors of $x^2 + ax + b$ are $(x - 7)$ and $(x + 9)$ then find the value of a and b

30. If $x + \frac{1}{x} = 7$, then find the value of $x^2 + \frac{1}{x^2}$.

LONG ANSWER TYPE

31. Find value of : $(a + b)^2 + (b + c)^2 + (c + a)^2$, when $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = 1$

32. If the speed of a bus is x km/hr in first hour, $(x + 2y)$ km/hr in second hour and $(2x - y)$ km/hr in the third hour. Then find the distance travelled by the bus in three hours.

33. A man's monthly income is Rs. $10x^3 + 5x^2 + 7x + 3$. He spend Rs. $3x^3 + 2x^2 + 2x + 1$ and Rs. $x^3 + 2x^2 + 3x + 1$ on education of his children and house rent respectively. Find the money left with him.

34. Solve : $(10x + 3y)(2x^2 + 5y)$.

35. Divide : $-15x^3y^4z^5 + 10x^2y^3z^4 - 25x^4y^3z^5$ by $-5x^2y^3z^3$.

36. If $2x+3y=13$ and $xy=5$ then find the value of $(2x-3y)^2$

37. Find the value of $(2017^2-2016^2)+(2017^2+2016^2+4034x2016) - (6033^2+2000^2-6033x4000)$

EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

1. The product of the reciprocal of $\frac{x+3}{x+2}$ and $\frac{x^2-4}{x^2-9}$ is

(A) $\frac{1}{(x-3)(x-2)}$ (B) $\frac{x-2}{x-3}$ (C) $\frac{x-3}{x-2}$ (D) $(x-3)(x-2)$

2. Simplify : $85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$.

(A) $44 + 104x$ (B) $-44 + 104x$ (C) $44 - 104x$ (D) $-44 - 104x$

3. Simplify : $-3(a + b) + 2(2a - b) + 4a - 5$.

(A) $5(a - b + 1)$ (B) $5(a + b + 1)$ (C) $5(a + b - 1)$ (D) $5(a - b - 1)$

4. Simplify : $xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}]$.

(A) $xy + 2zx + 3y$ (B) $xy - 2zx - 3y$ (C) $3xy + 2zx - 3y$ (D) $xy + 2zx - 3y$

5. If $\frac{a}{b} + \frac{b}{a} = 4$, find the value of $\frac{a^2}{b^2} + \frac{b^2}{a^2}$

(A) 16 (B) 18 (C) 14 (D) 20

6. If $a + \frac{1}{a} = 2$, then $a - \frac{1}{a}$ will be :

(A) 0 (B) 4 (C) – 2 (D) 2

7. a – b + [c – (a – b) + c – 2a]

(A) 2c + 2a (B) 2c – 2a (C) 3c – 2a (D) 3c + 2a

8. ab – [3ab – bc – (2bc – 3ac) + 2ac]

(A) – 2ab + 3bc – 5ac (B) – 2ab – 3bc – 5ac

(C) 2ab + 3bc – 5ac (D) – 2ab + 3bc +5ac

9. Reduced to lowest terms, $\frac{a^2-b^2}{ab} - \frac{ab-b^2}{ab-a^2}$ is equal to :

(A) $\frac{a}{b}$ (B) $\frac{a^2-2b^2}{ab}$ (C) a^2 (D) a – 2b

SECTION -B (TECHIE STUFF)

10. Find the remainder when $-4x + 4 - 4x^3 + x^4$ is divided by $2x - 2 + x^2$.

(A) 12x + 24 (B) – 44x + 32 (C) 20x + 24 (D) – 20x + 32

11. If $a - b = \frac{1}{3}$, then the value of $(a^3 - b^3 - ab)$ is ?

(A) 27 (B) $\frac{1}{27}$ (C) –27 (D) 0

12. The value of $\frac{(4.7)^3-(2.7)^3}{(4.7)^2+4.7\times2.7+(2.7)^2}$ is :

(A) 2 (B) 7.4 (C) 5 (D) 84.14

13. If x – y = 1, then the value of $x^3 - y^3 - 3xy$ will be :

(A) 1 (B) – 1 (C) 3 (D) – 3

EXERCISE 03

PREVIOUS YEAR EXAMINATION QUESTIONS

1. Given $a = 1\frac{5}{7}$, $b = \frac{1}{4}$, $c = \frac{1}{9}$ $d = \left(-1\frac{1}{4}\right)$, identify the value of a(b-c)+d **[NSTSE 2014]**

(A) $\frac{-4}{21}$ (B) $\frac{-6}{23}$ (C) $\frac{4}{21}$ (D) none of these

2. If x +y = 5 , x + z = 7 and y + z = 12 , then the value of x +y +z is : **[NSTSE 2010]**

(A) 12 (B) 2 (C) 5 (D) 24

3. For what value of 'x' does the equation $\frac{a+b-x}{c} + \frac{a+c-x}{b} + \frac{c+b-x}{a} + \frac{4x}{a+b+c} = 1$ satisfy.

[NSTSE 2011]

(A) ab + bc + ca (B) 0 (C) a + b + c (D) 1

4. If ab = 1, then $\frac{1}{1+a^{-1}}+\frac{1}{1+b^{-1}}$ is : **[NSTSE 2011]**

(A) 0 (B) a + b (C) 1 (D) a – b

5. $1+\frac{1}{x}=\frac{x+1}{x}$, what does x equal to ? **(NSTSE 2010)**

(A)1 or 2 only (B)1 and 0 only

(C) 1 and -1 only (D) any number except 0

6. Simplify : [(31 – 19) × (5 – (5 + 2 – 3)} of 3 + (– 2)] × (–1) **[IMO-2012]**

(A) 24 (B) – 34 (C) – 54 (D) 12

7. Mohit spends Rs.7x – 4 for a shirt and Rs.2x + 8 for a pair of trousers. If he gives the shopkeeper a 1000 rupee note, how much will he get back? **[IMO-2013]**

(A) 1012– 9x (B) 996 – 9.x (C) 992 – 8x (D) 996 — 7x

8. In a school, $8a^2 + 4a + 9$ students were enrolled. $2a^2 – 9a + 2$ students were boys. How many girls were enrolled? **[IMO-2013]**

(A) $6a^2–13a+7$ (B) $4a^2 + 13a + 7$ (C) $6a^2 + 13a + 7$ (D) $4a^2—13a + 7$

9. Simplify, the expression by removing brackets. **[IMO-2013]**

$$a-2b-\left[4a-6b-\left\{3a-c+\left(5a-2b-\overline{3a-c+2b}\right)\right\}\right]$$

(A) 5a + b (B) 4a - c (C) 3a + b+ c (D) 2a

ANSWER KEY

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	D	B	C	B	C	B	A	A	C	B	D	C
Ques.	16	17	18	19											
Ans.	A	B	D	C											

FILL IN THE BLANKS

1. trinomial **2.** variable **3.** 0 **4.** –7 **5.** divisor

6. $7x - 4y$ **7.** $(x^2 - 4)$ **8.** $a^2 + 6a + 9$

TRUE / FALSE

1. False **2.** True **3.** True **4.** True **5.** False

6. True **7.** False **8.** False

MATCH THE COLUMN

1. (A) → s, (B) → q, (C) → r, (D) → t, (E) → p, (F) → w, (G) → u, (H) → v, (I) → x

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. **(i)** Binomial, linear **(ii)** Trinomial cubic

(iii) Binomial, linear **(iv)** monomial, constant

(v) monomial, linear.

2. **(i)** 12, 7, – 1 **(ii)** 5, 7, 8 **(iii)** – 1, 1 , 8, 7 **(iv)** 1, 9

3. – 7 **4.** $2x^2$, 3y, – 5x, 4 **5.** –13ab **6.** 10x – 1

7. – 10xyz **8.** $15x^2y^3z$ **9.** $2x^3 + 2xy$ **10.** $-115x^2 - 23xy$

11. $4x^2$ **12.** –9xy **13.** $x^3 + 4x^2 - 3$ **14.** $-2x^2 + 8x - 4$

15. $x^2 - y^2$ **16.** 16 **17.** 10

SHORT ANSWER TYPE

18. **(i)** P – Q **(ii)** (P + Q) – PQ **(iii)** 20x + 10y

(iv) $\frac{x}{y} = z$ **(v)** $\frac{3}{4}x + \frac{2}{5}y = 18$.

19. **(i)** sum of x, y and z

(ii) product of sum of x and y, and difference of x and y.

(iii) two times x in added to three times y

(iv) two times x in divided by five times y gives quotient seven.

(v) sum of two third of x, one fifth of y, seven by two of z.

20. **(i)** $3xy^2, xy^2, -5xy^2$ **(ii)** $-3x^2, 7x^2$ **21.** $(5x^5 + 5x^4 + 4x^3 + 4x^2 + x + 4)$

22. $-x^4 + 4x^3 - 3x - 3$ **23.** $3p^4 - p^3 - 9p^2 + 6p - 2$.

24. **(i)** 4 **(ii)** 4 **(iii)** – 30 **(iv)** – 20

(v) 1 **(vi)** 3 **(vii)** 5

25. $x^2 - 2x + 4$ **26.** $3b^2c^3 + 7a^5b^2 - 5a^2bc^4$ **27.** 4

28. **(i)** $9x^2+49y^2-42xy$ **(ii)** $16a^2+9b^2c^2+24abc$ **29.** a = 2, b = – 63

30. 47

LONG ANSWER TYPE

31. $4\frac{13}{18}$ **32.** 4x + y **33.** $6x^3 + x^2 + 2x + 1$

34. $20x^3 + 6x^2y + 50xy + 15y^2$ **35.** $3xyz^2 - 2z + 5x^2z^2$ **36.** 49

37. 4033

EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9
Ans.	C	A	D	D	C	A	B	A	A

SECTION -B (TECHIE STUFF)

Ques.	10	11	12	13
Ans.	B	B	A	A

EXERCISE 03

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9
Ans.	D	A	C	C	D	B	B	C	D