MATHEMATICS

Class-VII

Topic-12

SYMMETRY & VISUALIZING SOLID SHAPES

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CHAPTER 12

SYMMETRY & VISUALIZING SOLID SHAPES

TERMINOLOGIES

Symmetry, Line of Symmetry, Axis of Symmetry, Rotational Symmetry, Order of Ratational Symmetry, Prism, Pyramid, Sphere, Euler's Fomrula, Cuboid, Cube, Triangular Pyramid, Cylinder, Cone, Net.

INTRODUCTION

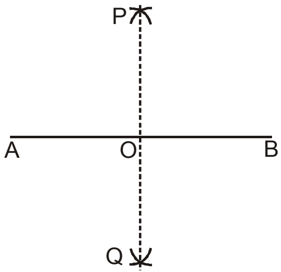
Symmetry is an important geometrical concept, commonly exhibited in nature, Artist, professionals, designers of clothing or jewellery, car manufacturers all uses the idea of symmetry.
Symmetry abounds in many naturally occuring objects for example, if we start from our body we can see our hands (length), legs (length) with evenly balanced proportion, we say, "they are symmetrical". Other naturally occurring objects involving the concept of symmetry are : The flowers, the tree leaves etc. so lets study it in detail.

12.1 SYMMETRY

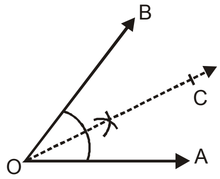
❖ If a line divides a figure into two parts such that when the figure is folded about the line, the two parts of the figure coincide, then the line is known as the **line of symmetry.** The line of symmetry is also known as the **axis of symmetry.**

For example :

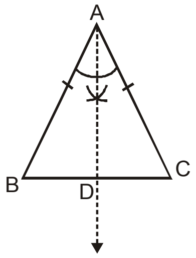
1. A line segment is symmetrical about its perpendicular bisector as shown :



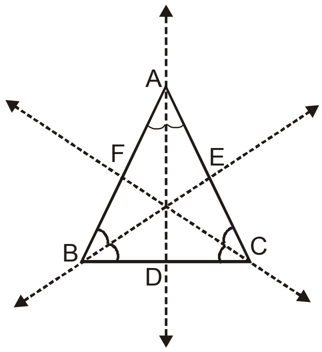
2. A given angle having equal arms is symmetrical about bisector of the angle.



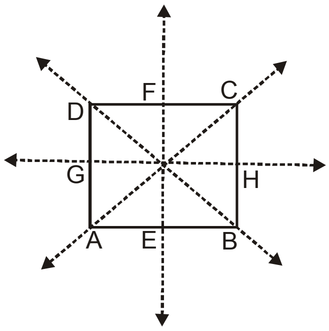
3. An isosceles triangle is symmetrical about the bisector of the angle included between the equal sides.



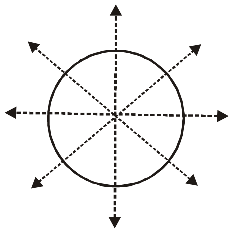
4. An equilateral triangle is symmetrical about each one of the bisector of its interior angles.



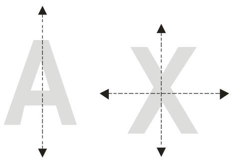
5. A square has 4 lines of symmetry, namely the diagonals and the lines joining the midpoints of its opposite sides.



6. A circle is symmetrical about each one of its diameters.



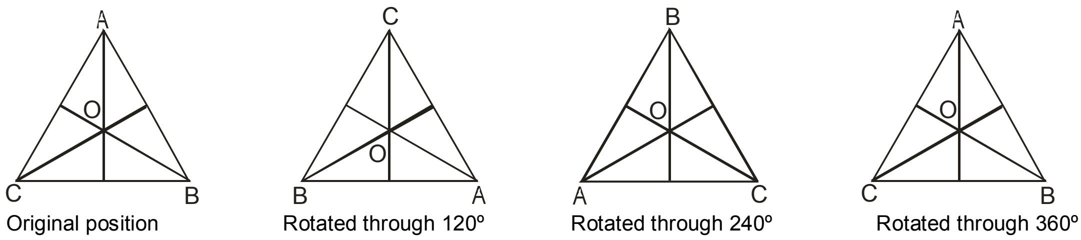
7. Each of the following capital letters of the English alphabet is symmetrical about the dotted line or lines as shown :



❖ A figure is said to have **rotational symmetry,** if it fits on to itself more than once during a full turn i.e. rotation through 360°.

❖ The number of times a figure fits onto itself in one full turn is called the **order of rotational symmetry**.

Figure, below shows an equilateral triangle whose medians intersect at O. If we rotate the triangle through 120°, 240° and 360°, the triangle does not appear to have moved, unless we label the vertices A, B and C.



There are three positions where it appears not to have moved. We say that an equilateral triangle has rotational

Illustration 12.1

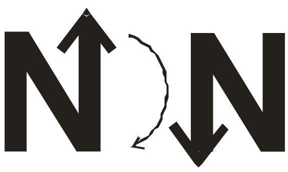
Which of the following letters have rotational symmetry ?

D H I M N U

Sol. The letters H, I and N have rotational symmetry.

Original position   Rotated through 180°

Original position   Rotated through 180°



Original position   Rotated through 180°

❖ Following table provides the details of linear and rotational symmetries of various figures :

Figure	**Line Symmetry**	**No. of Line Symmetry**	**Rotational Symmetry**	**Centre of Rotation**	**Order**
Square	Yes	4	Yes	Intersection of diagonals	4
Rectangle	Yes	2	Yes	Intersection of diagonals	2
Equilateral Triangle	Yes	3	Yes	Centroid	3
Regular Hexagon	Yes	6	Yes	Centre of the hexagon	6
Circle	Yes	**Unlimited**	Yes	Centre	**Unlimited**
Rhombus	Yes	2	Yes	Intersection of diagonals	2

Ask yourself

1. State the number of lines of symmetry for the following figures:

(i) Rhombus **(ii)** Regular pentagon

2. What other name can you give to the line of symmetry of:
(i) An isosceles triangle **(ii)** A circle

3. Identify three examples of shape with no line of symmetry.

4. Name any two figures that have both line symmetry and rotational symmetry.

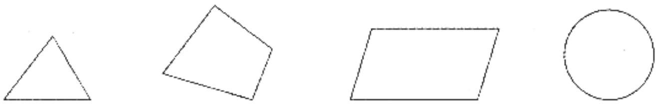
5. Determine the order of rotational symmetry of an equilateral triangle.

Answers

1. **(i)** 2 **(ii)** 5 **2.** **(i)** Altitude on base **(ii)** Diameter
3. Scalene triangle, Parallogram, Trapezium **4.** Equilateral triangle, Rectangle
5. 3

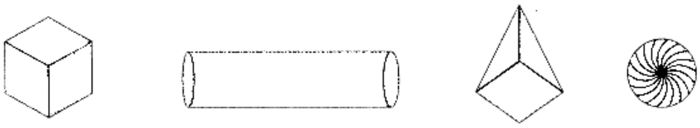
12.2 VISUALIZING SOLID SHAPES

The figures such as triangles, rectangles, hexagons, squares, circles etc., which have only two dimensions namely length and breadth are called plane figures or 2-dimensional figures.



Plane figures (two dimensional figures)

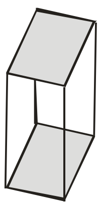
The figures such as cube, cuboid, cylinder, pyramid, etc., which have three dimensions, namely length, breadth and height are called solid figures or 3-dimensional figures.



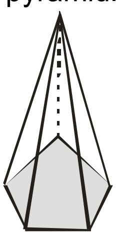
Solid figures (Three dimensional figures)

(a) Types of solid

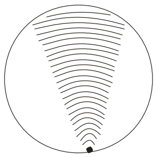
There are mainly three types of solids :
(i) Prism: A solid whose base and top are identical polygons and side faces are rectangles, is called a **prism**. In a square prism whose base and top are congruent squares.



(ii) Pyramid : A solid whose base is any polygon and side faces are triangles, is called a **pyramid**. Figure shows a pentagonal pyramid.



(iii) Sphere : Sphere is a solid whose every point is equidistant from a fixed point. Figure shows the sphere.



NOTE:

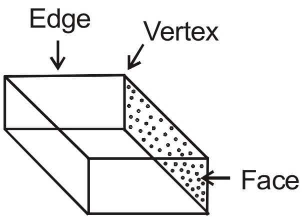
(i) All the side faces of a pyramid (triangular, rectangular, squares, pentagonal etc.) are triangular.

(ii) A pyramid is named according to the shape of its non-triangular face. If all the faces are triangular, then it is called a triangular pyramid or tetrahedron.

(b) Euler's formula

The number of faces, edges and vertices of prism and pyramids are connected by the formula V + F – E = 2, where V, F and E stand for the number of vertex, face and edge. This formula is known as **Euler's formula.**

Cuboid :

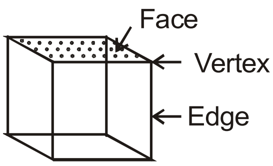


A cuboid has

(i) 6 rectangular faces **(ii)** 12 edges **(iii)** 8 vertices.

Let F, E and V denote respectively the number of faces, edges and vertices of a cuboid.
Then, F – E + V = 6 – 12 + 8 = 2.

Cube (Square prism) :



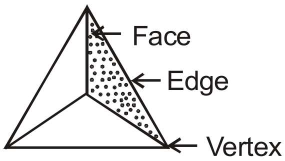
A cube has

(i) 6 square faces **(ii)** 12 edges **(iii)** 8 vertices

Here, F = 6, E = 12 and V = 8

$\therefore$ F – E + V = 6 – 12 + 8 = 2.

Triangular Pyramid : A triangular pyramid (Tetrahedron) shown in fig. has :



(i) 4 faces **(ii)** 6 edges **(iii)** 4 vertices

Here, F = 4, E = 6 and V = 4

$\therefore$ F – E + V = 4 – 6 + 4 = 2

Illustration 12.2

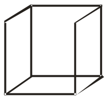
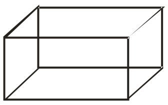
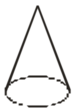
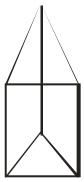
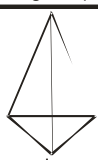
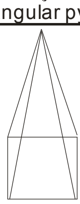
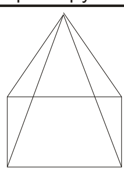
A polyhedron has 30 edges and 20 vertices. How many faces does the polyhedron have ?

Sol. Here, E = 30, V = 20

$$V + F - E = 2 \quad \Rightarrow \quad 20 + F - 30 = 2 \quad \Rightarrow \quad F = 2 - 20 + 30 = 12$$

Hence, the polyhedron has 12 faces.

❖ Following table provides the details of the number of faces, edges and vertices of some solids.

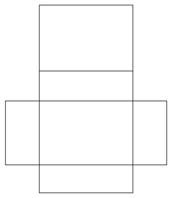
Solid	No. of faces	No. of vertices	No. of edges
 Cube	6	8	12
 Cuboid	6	8	12
 Cylinder	3	—	2
 Cone	2	1	1
 Sphere	1	–	–
 Triangular prism	5	6	9
 Triangular pyramid	4	4	6
 Square pyramid	5	5	8
 Rectangular pyramid	5	5	8

(c) Net for building 3-d shapes

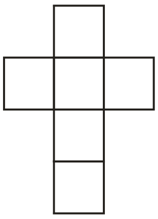
In order to understand three dimensional objects more closely, we try to form these objects from their nets. A net for a three dimensional shape is nothing but a sort of skeleton-outline in 2-dimension which, when folded, results in three dimensional shape.

Following are net patterns for different three dimensional shapes :

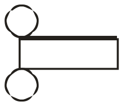
(i) Net pattern for a cuboid :



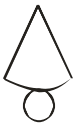
(ii) Net pattern for cube :



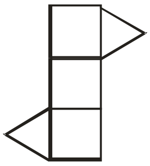
(iii) Net pattern for a cylinder :



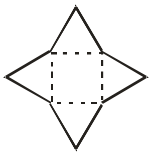
(iv) Net pattern for a cone :



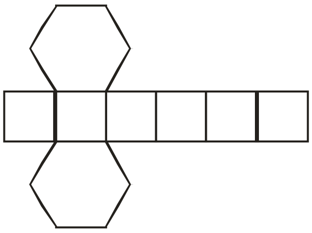
(v) Net pattern for a triangular prism :



(vi) Net pattern for a square pyramid :



(vii) Net pattern for a hexagonal prism :



Ask yourself

1. How many faces, vertices and edges are there in cube, cuboid and cylinder.
2. For the following figures verify the Euler's formula :
   (i) Cuboid **(ii)** Triangular Pyramid **(iii)** square Pyramid.
3. Which of the following are 2 -D figures and which are 3-D figures
   (i) rectangle **(ii)** cylinder **(iii)** circle
   (iv) sphere **(v)** octagon **(vi)** cone
4. Draw a net for open cylinder.
5. From your surroundings,give two examples each of the following shapes :
   (i) cube **(ii)** cuboid **(iii)** cone
   (iv) cylinder **(v)** sphere.

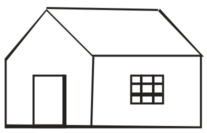
Answers

1. Cube - 6, 8, 12, Cuboid – 6, 8, 12, Cylinder – 3, 0, 2
3. 2D → rectangle, circle, octagon 3D → circle, sphere, cone
5. **(i)** Rubical cube, Cubical box **(ii)** Book, Room
   (iii) Birthday cap, Ice cream cone **(iv)** Cake, Pipe **(v)** Cricket ball, Football

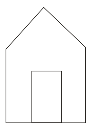
Add your knowledge

(f) View of 3D-Shapes

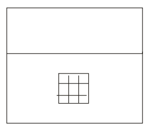
As we know that a 3-dimensional object can look differently from different positions so they can be drawn from different perspectives. For example, a given hut can have the following views.



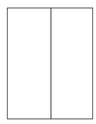
Object



Front view (from right)

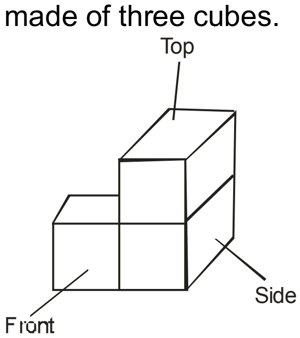


Side view

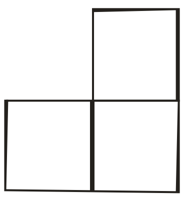


Top view

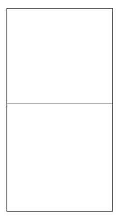
We can also get different views of figures made by joining cubes. For example an object made of three cubes.



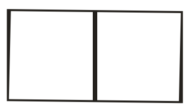
Object



Front view (from right)

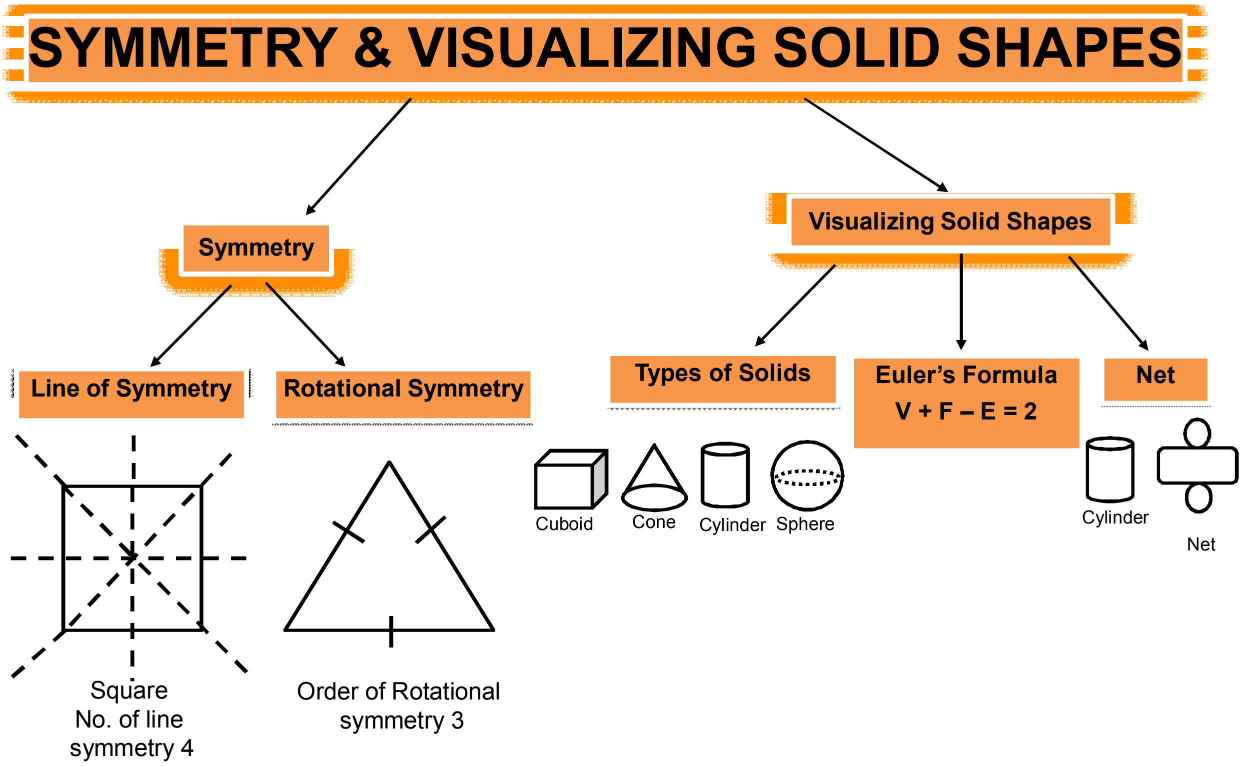


Side view



Top view

Concept Map


SYMMETRY & VISUALIZING SOLID SHAPES
Symmetry
Visualizing Solid Shapes
Line of Symmetry
Rotational Symmetry
Types of Solids
Euler's Formula
V + F – E = 2
Net
Cuboid
Cone
Cylinder
Sphere
Cylinder
Net
Square
No. of line
symmetry 4
Order of Rotational
symmetry 3

Summary

1. If a line divides a figure into two parts such that when the figure is folded about the line, the two parts of the figure coincide, then the line is known as the **line of symmetry.**

2. A line segment is symmetrical about its perpendicular bisector

3. An isosceles triangle is symmetrical about the bisector of the angle included between the equal sides.

4. An equilateral triangle is symmetrical about each one of the bisector of its interior angles.

5. A square has 4 lines of symmetry, namely the diagonals and the lines joining the midpoints of its opposite sides.

6. A circle is symmetrical about each one of its diameters.

7. A figure is said to have **rotational symmetry,** if it fits on to itself more than once during a full turn i.e. rotation through 360°.

Figure	Line Symmetry	No. of Line Symmetry	Rotational Symmetry	Centre of Rotation	Order
Square	Yes	4	Yes	Intersection of diagonals	4
Rectangle	Yes	2	Yes	Intersection of diagonals	2
Equilateral triangle	Yes	3	Yes	Centroid	3
Regular Hexagon	Yes	6	Yes	Centre of the hexagon	6
Circle	Yes	**Unlimited**	Yes	Centre	**Unlimited**
Rhombus	Yes	2	Yes	Intersection of diagonals	2

9. All the side faces of a pyramid (triangular, rectangular, squares, pentagonal etc.) are triangular.

10. A pyramid is named according to the shape of its non-triangular face. If all the faces are triangular, then it is called a triangular pyramid or tetrahedron.

11. **Euler's formula.:** V + F – E = 2,
V, F and E stand for the number of vertex, face and edge.

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

1. The lines of symmetry in a square are :
(A) 2 (B) 1 (C) 4 (D) 3

2. The lines of symmetry in a regular hexagon are :
(A) 2 (B) 1 (C) 4 (D) 6

3. The lines of symmetry in an isosceles triangle are :
(A) 1 (B) 2 (C) 3 (D) 4

4. An equilateral triangle has rotational symmetry of order :
(A) 2 (B) 1 (C) 4 (D) 3

5. A square has rotational symmetry of order :
(A) 2 (B) 3 (C) 4 (D) 1

6. Circle is a :
(A) plane figure (B) solid figure (C) both (D) none ofthese

7. The other name of a tetrahedron is :
(A) triangular pyramid (B) triangular prism
(C) square pyramid (D) none of these

8. A pentagonal pyramid has :
(A) 3 vertices (B) 4 vertices (C) 6 vertices (D) none of these

9. A square prism has :
(A) 5 edges (B) 8 edges (C) 12 edges (D) 15 edges

10. A cylinder has :
(A) 1 face (B) 2 faces (C) 3 faces (D) 5 faces

11. A rectangular pyramid has :
(A) 2 faces (B) 4 faces (C) 5 faces (D) 6 faces

12. The name of the figure which has 6 vertices, 9 edges and 5 faces is :
(A) cuboid (B) cube (C) cone (D) triangular prism

13. Name the solid figure which has no vertex and no edge :
(A) cylinder (B) cone (C) sphere (D) tetrahedron

14. The net of a solid consists of three rectangles and two triangles. This may be the net of a :
(A) cuboid (B) pyramid (C) triangular prism (D) none of these

15. The net for a cylinder without top and bottom is a :
(A) rectangle (B) circle (C) triangle (D) none of these

FILL IN THE BLANKS

1. The number of positions in which figure looks exactly the same after a rotation is called the ______.
2. The number of line symmetry of a letter H is____
3. Cuboid has _________ number of vertices
4. Triangular prism has _________ faces
5. A regular polygon of n sides has ___________ lines of symmetry.
6. V + F – E = 2 is known as _________
7. Rectangular pyramid has _____ number of edges
8. Sphere has __________ faces.

TRUE / FALSE

1. A circle has an infinite number of lines of symmetry.
2. An equilateral triangle has no line of symmetry.
3. A parallelogram has 2 lines of symmetry.
4. A semicircle has one line of symmetry.
5. In prism, base and top are different polygon.
6. Number of faces in cuboid is 6

MATCH THE COLUMN

1.

	Column–I		Column–II
(A)	Lines of symmetry in Rhombus	(p)	infinite
(B)	lines of symmetry in regular hexagon	(q)	one
(C)	lines of symmetry in regular pentagon	(r)	two
(D)	lines of symmetry in isosceles triangle	(s)	six
(E)	lines of symmetry in circle	(t)	five

SECTION -B (FREE RESPONSE TYPE)

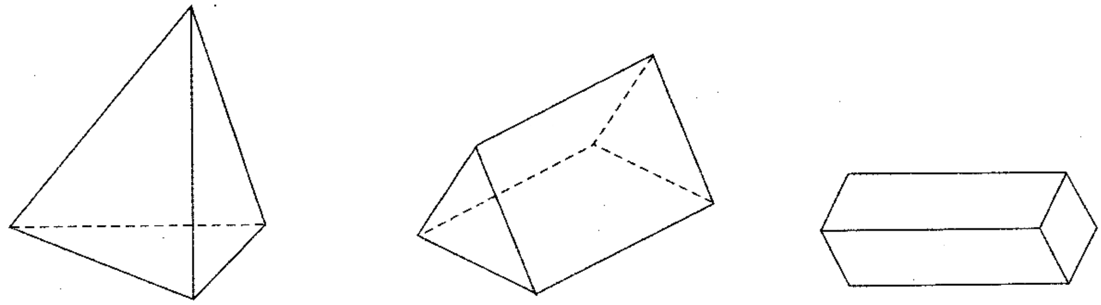
VERY SHORT ANSWER TYPE

1. Which of the following letters of English alphabet have both line symmetry and rotational symmetry ?

A C I M N O Z

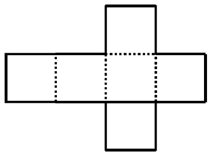
2. Does every trapezium has a line of symmetry?

3. Name the following solid shapes.

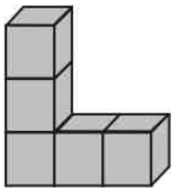


(i) (ii) (iii)

4. Look at the given net (flat pattern). If the net (flat pattern) is folded, which geometric figure will it be



5. How many total numbers of cubes have been used to build the given object

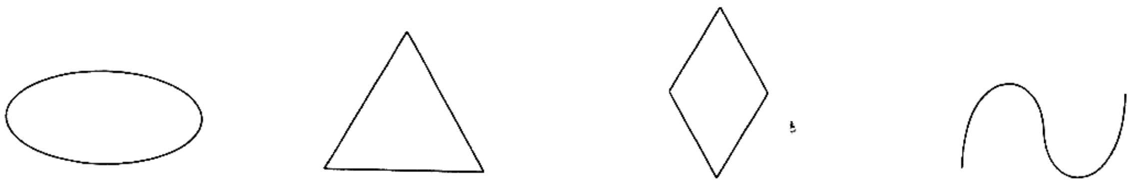


SHORT ANSWER TYPE

6. For each of the following shapes state :

(a) The number of lines of symmetry.

(b) The order of rotational symmetry.



(i) (ii) (iii) (iv)

7. Discuss rotational symmetry of a square . Also determine its line of symmetry.

8. Give an example of a geometrical figure which has neither a line of symmetry nor rotational symmetry.

9. Give the geometrical name for each of the following :

(a) book **(b)** die **(c)** fridge **(d)** football

(e) unsharpened pencil

10. Write down the number of vertices in each of the following solids:

(a) Cube **(b)** Cone **(c)** Tetrahedron

(d) Triangular prism **(e)** Rectangular pyramid

11. Write down the number of edges in each of the following solids :

(a) Cuboid **(b)** Sphere **(c)** Cylinder

(d) Triangular pyramid

12. A polyhedron has 30 edges and 20 vertices. How many faces does the polyhedron have?

LONG ANSWER TYPE

13. Draw, wherever possible, a rough sketch of

(a) a triangle with both line and rotational symmetries of order more than 1

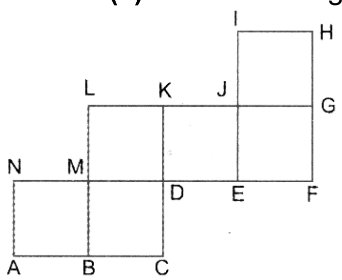
(b) a triangle with only line symmetry and no rotational symmetry of order more than 1.

(c) a quadrilateral with no line symmetry and rotational symmetry of order more than 1.

(d) a quadrilateral with line symmetry but not a rotational symmetry of order more than 1.

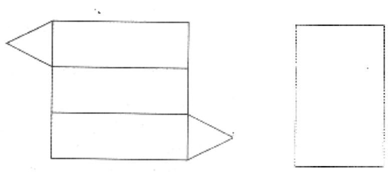
14. The given net can be used to make a cube.

(i) Which edge meets AN ? **(ii)** Which edge meets DE ?



15. Draw a net for a triangular prism whose ends are equilateral triangles.

16. The diagrams given below show the nets of various solids. Name the solids.

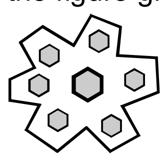


EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

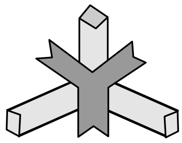
MULTIPLE CHOICE QUESTIONS

1. The number of lines of symmetry in the figure given below is :



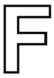
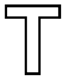
(A) 4 (B) 8 (C) 6 (D) Infinitely many

2. The number of lines of symmetry in figure is

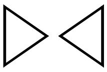
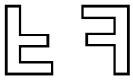
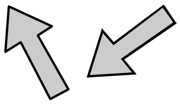
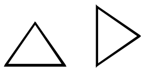


(A) 1 (B) 3 (C) 6 (D) Infinitely many

3. Which of the following has a line of symmetry ?

(A)  (B)  (C)  (D) 

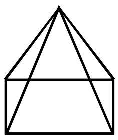
4. Which of the following are reflections of each other ?

(A)  (B)  (C)  (D) 

5. Which of the following letters of English alphabets have more than 2 lines of symmetry ?

(A) Z (B) O (C) E (D) H

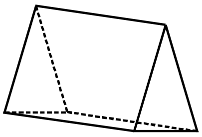
6. The name of the given solid in figure is



(A) triangular pyramid (B) rectangular pyramid

(C) rectangular prism (D) triangular prism

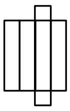
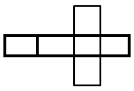
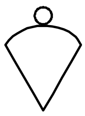
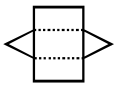
7. The name of the solid in figure is



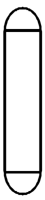
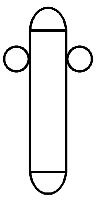
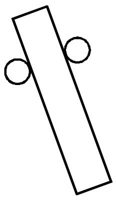
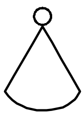
(A) triangular pyramid (B) rectangular prism

(C) triangular prism (D) rectangular pyramid

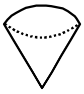
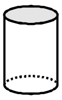
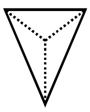
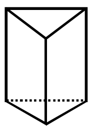
8. Which of these nets is a net of a cube ?

(A)  (B)  (C)  (D) 

9. Which of the following nets is a net of a cylinder ?

(A)  (B)  (C)  (D) 

10. Which of the following 3-dimensional figures has the top, side and front as triangles ?

(A)  (B)  (C)  (D) 

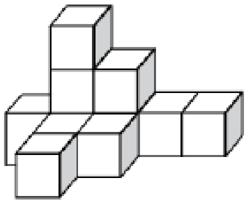
11. All faces of a pyramid are always
(A) Triangular (B) Rectangular (C) Congruent (D) None of these

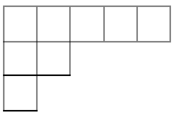
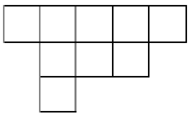
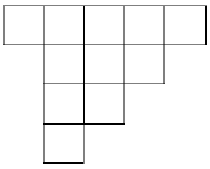
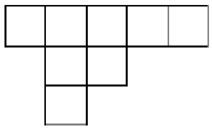
12. A solid with one circular face, one curved surface and one vertex
(A) cone (B) sphere (C) cylinder (D) prism

SECTION -B (TECHIE STUFF)

Direction: (For Q.no. 13 and 14)

13. The diagram given shows a three-dimensional object made of 11 identical cubes. Which of the following best represents the top view of this three-dimensional object?



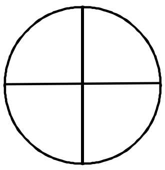
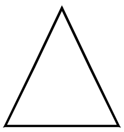
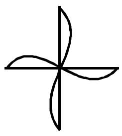
(A)  (B)  (C)  (D) 

14. Find the no of the cubes ?
(A) 8 (B) 9 (C) 10 (D) 11

EXERCISE 03

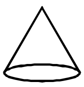
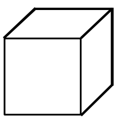
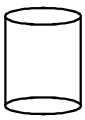
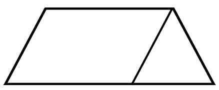
(PREVIOUS YEAR EXAMINATION QUESTIONS)

1. Which of the following figure have rotational symmetry of order more than 1 ? **[NSTSE 2010]**

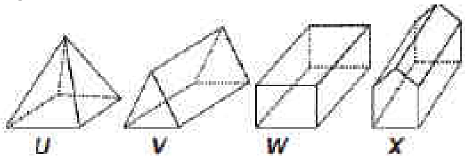
(A)  (B)  (C)  (D) All the given

2. Choose the correct statement from the following : **[NSTSE 2010]**
(A) A triangle has 3 sides and 4 vertices
(B) a cylinder has 3 faces
(C) All sides of the rectangle are equal
(D) A cuboid has 4 flat faces and 12 straight edges.

3. Which of the following figure has six faces ? **[NSTSE 2010]**

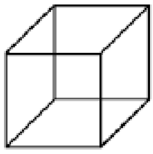
(A)  (B)  (C)  (D) 

4. Which of the following figures has 10 vertices? **[IMO-2010]**



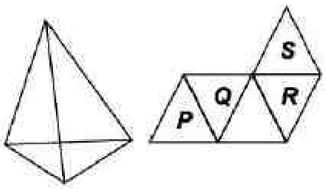
(A) Figure U (B) Figure V (C) Figure W (D) Figure X

5. The number of edges in the given figure is __________. **[IMO-2011]**



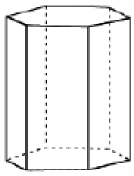
(A) 8 (B) 9 (C) 10 (D) 12

6. The given solid (not drawn to scale) is a regular triangular pyramid. The net of the pyramid is wrongly drawn with 1 extra face. Which of the following faces (P, Q, R or S) is NOT a part of the net? **[IMO-2012]**



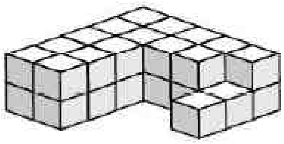
(A) P (B) Q (C) R (D) S

7. How many faces does the given solid have? **[IMO-2012]**



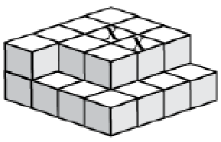
(A) 8 (B) 10 (C) 12 (D) 14

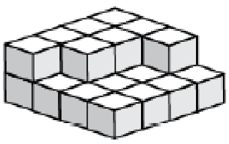
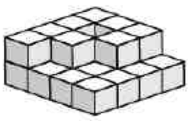
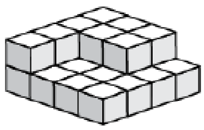
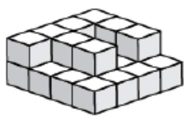
8. How many unit cubes are required to make it a cuboid? **[IMO-2013]**



(A) 18 (B) 30 (C) 28 (D) 26

9. If the 2 unit cubes marked 'X' are removed from the solid, which solid will be obtained? **[IMO-2013]**

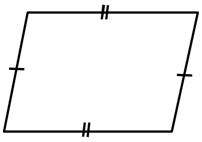
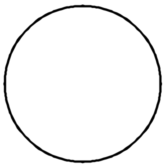
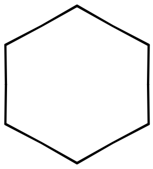
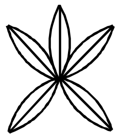


(A)  (B)  (C)  (D) 

10. Select the INCORRECT match. **[IMO-2013]**

	Faces	**Edges**	**Vertex**
(A) Cube	6	12	8
(B) Cylinder	1	1	1
(C) Triangular Pyramid	4	6	4
(D) Cuboid	6	12	8

11. Identify the figure with more than 6 lines of symmetry **[NSTSE 2014]**

(A)  (B)  (C)  (D) 

12. Figure (X) has rotational symmetry of order. **[IMO-2014]**

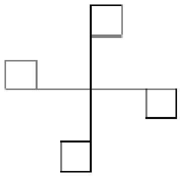


Figure (X)

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER KEY

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	D	A	D	C	A	A	C	C	C	C	D	C	C	A

FILL IN THE BLANKS

1. order of rotational symmetry **2.** 2 **3.** 8 **4.** 5

5. n **6.** Euler's formula **7.** 8 **8.** 1

TRUE / FALSE

1. True **2.** False **3.** False **4.** True **5.** False

6. True

MATCH THE COLUMN

1. (A) $\to$ r, (B) $\to$ s, (C) $\to$ t, (D) $\to$ q, (E) $\to$ p

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. **I** and **O** **2.** No

3. **(i)** Triangular pyramid **(ii)** Triangular prism **(iii)** Rectangular prism / cuboid

4. cube **5.** 5

SHORT ANSWER TYPE

6. **(i)** (a) 2, (b) 2, **(ii)** (a) 3, (b) 3 **(iii)** (a) 2, (b) 2, **(iv)** (a) 0, (b) 2

7. 4, 4 **8.** Scalene triangle

9. **(a)** cuboid **(b)** cube **(c)** cuboid **(d)** sphere
(e) cylinder or prism

10. **(a)** 8 **(b)** 1 **(c)** 4 **(d)** 6 **(e)** 5

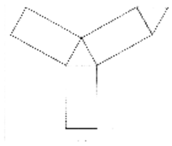
11. **(a)** 12 **(b)** none or 0 **(c)** 2 **(d)** 6

12. 12

LONG ANSWER TYPE

13. **(a)** equilateral triangle **(b)** Isosceles triangle
(c) Parallelogram **(d)** isosceles trapezium

14. **(i)** GH **(ii)** CD

15. 

16. **(i)** Triangle prism **(ii)** Cylinder with both ends open

EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	C	B	C	A	B	B	C	B	C	C	D	A

SECTION -B (TECHIE STUFF)

Ques.	13	14
Ans.	D	D

EXERCISE 03

(PREVIOUS YEAR EXAMINATION QUESTIONS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	D	B	B	D	D	D	A	A	D	B	B	D