MATHEMATICS

Class-VIII

Topic-7

PLAYING WITH NUMBERS

INDEX

CH-07

PLAYING WITH NUMBERS

TERMINOLOGIES

Divisibility test, alphanumeric puzzles, generalized form, ten's digit, unit digit.

INTRODUCTION

In previous classes, we have learnt about various types of numbers for example natural number, whole number, integers, rational numbers. Earlier, Mathematics was known as the difficult and boring subject. But different activities, puzzles and games have made it easier and and interesting, different concepts and ideas helped the students to observe pattern, explore then and try to define them in their own way.

7.1 PLAYING WITH NUMBERS

(a) Number in general form

Any number is said to be in a generalized form if it is expressed as the sum of the product of its digits with their respective place values.

Thus, a two-digit number having a and b as its digits at the tens and the ones places respectively is written in the generalized form as 10a + b, i.e., in general, a two-digit number can be written as 10 a + b, where 'a' can be any of the digits from 1 to 9 and 'b' can be any of the digits from 0 to 9.

Similarly, a three-digit number can be written in the generalized form as 100 a + 10 b + c, where 'a' can be anyone of the digits from 1 to 9 while 'b' and 'c' can be any of the digits from 0 to 9.

The generalized forms of a few numbers are given below:

56 =10 × 5 + 6 ;

90 = 10 × 9 + 0 ;

129 = 100 × 1 + 2 x 10 + 9 ;

206 = 100 x 2 + 10 x 0 + 6 ;

Illustration 7.1

If the number obtained by interchanging the digits of a two-digit number is 18 more than the original number and the sum of the digits is 8 then what is the original number?

Sol. Let the original number be 10a + b. Then, 'a' is the tens digit and 'b' is the units digit. Since the sum of the digits is 8, therefore a + b = 8, i.e., b = 8 – a .

So, the original number is 10a + (8 – a).

∴ the number obtained by interchanging the digits is 10(8 – a) + a, and so we have

{l0(8 – a) + a} – {10a + (8 – a)} = 18.

Solving this equation, we get

a = 3.

And so, b = 8 – a = 8 – 3 = 5.

Hence, the original number is 10a + b = 30 + 5 = 35.

Illustration 7.2

In a two-digit number, the digit in the units place is four times the digit in the tens place and sum of the digits is equal to 10. What is the number?

Sol. Let the original number be 10a + b.
Then, b = 4a and a + b = 10.
We put b = 4a in a + b = 10 so that a + 4a = 10, i.e., 5a = 10, i.e., a = 2.
∴ a = 2 and b = 4a = 8.
Hence, the number is 10a + b = 20 + 8 = 28.

(b) Divisibility test

Number	Divisiblity Test
2	Unit digit should be 0 or even
3	The sum of digits of no. should be divisible by 3
4	The no. formed by last 2 digits of given no. should be divisible by 4.
5	Unit digit should be 0 or 5.
6	No. should be divisible by 2 & 3 both
8	The number formed by last 3 digits of given no. should be divisible by 8.
9	Sum of digits of given no. should be divisible by 9
11	The difference between sums of the digits at even & at odd places should be zero or multiple of 11.
25	Last 2 digits of the number should be 00, 25, 50 or 75.

Illustration 7.3

Replace * by the smallest digit, so that 1*4 is divisible by (i) 3, (ii) 9. Find the number also.

Sol. **(i)** Sum of the digits of 1*4=1+4=5
The number next to 5 which is divisible by 3 is 6, so * is to be replaced by 1
∴ The number becomes 114

(ii) Sum of the digits of 1*4=1+4=5
The number next to 5 which is divisible by 9 is 9, so * is to be replaced by 4
∴ The number becomes 144

Illustration 7.4

Replace 'a' by the smallest digit, so that 41a624 is divisible by 11. Find the number also.

Sol. Sum of the digits at odd places =4+6+1=11
Sum of the digits at even places =2+a+4=6+a
So, the difference of sum of digits at odd places and even places=0 or multiple of 11
⇒ 11– (6+a) = 0
⇒ 11– 6 –a = 0
⇒ 5 – a = 0
⇒ a = 5.

(c) Alpha Numeric Puzzles

In general, we have digits in any mathematical operations. But in these puzzles letters are given in place of digits and we have to code these letters to find out the corresponding digit for the letter. Some addition & multiplication puzzles are as follows :

Illustration 7.5

Find Q in the addition.

$$\begin{array}{r} 3\ 1\ Q \\ +\ 1\ Q\ 3 \\ \hline 5\ 0\ 1 \end{array}$$

Sol. As the addition in the ones column : From Q + 3, we get '1', that is, a number whose ones digit is 1.

For this to happen, the digit Q should be 8. So the puzzle can be solved as shown below :

$$\begin{array}{r} 3\ 1\ 8 \\ +\ 1\ 8\ 3 \\ \hline 5\ 0\ 1 \end{array}$$

So, Q = 8.

Illustration 7.6

Find A and B in the addition.

$$\begin{array}{r} A \\ +\ A \\ +\ A \\ \hline B\ A \end{array}$$

Sol. As the addition in the ones column : The sum of three A' s is a number whose one digit is A. Therefore, the sum of two A's must be a number whose ones digit is 0.

This happens only for A = 0 and A = 5.

If A = 0, then the sum is 0 + 0 + 0 = 0, which makes B = 0 too. We do not want this (as it makes A = B , and then the tens digit of BA too becomes 0), so we reject this possibility.

So, A = 5.

Therefore, the puzzle is solved as shown below.

$$\begin{array}{r} 5 \\ +\ 5 \\ +\ 5 \\ \hline 1\ 5 \end{array}$$

That is , A = 5 and B = 1.

Illustration 7.7

Find the digits A and B.

$$\begin{array}{r} B\ A \\ \times\ B\ 3 \\ \hline 5\ 7\ A \end{array}$$

Sol. This also has two letters A and B whose values are to be found.

Since the ones digit of 3 × A is A, it must be that A = 0 or A = 5.

Now look at B. If B = 1, then BA × B3 would at most be equal to 19 × 19; that is, it would at most be equal to 361. But the product here is 57A, which is more is more than 500. So we cannot have B = 1.

If B = 3, then BA × B3 would be more than 30 × 30; that is, more than 900. But 57A is less these two facts together, we see that B = 2 only. So the multiplication is either 20 × 23, or 25 × 23.

The first possibility fails, since 20 × 23 = 460. But, the second one works out correctly, since 25 × 23 = 575.

$$\begin{array}{r} 2\ 5 \\ \times\ 2\ 3 \\ \hline 5\ 7\ 5 \\ \hline \end{array}$$

So the answer is A = 5, B = 2.

Illustration 7.8

In the following problem, replace the letters of the English alphabet by digits [two or more letters may have the same value] to complete the procedure of division.

$$\begin{array}{r} 5C \\ 9\overline{)\,4\,A\,B} \\ -DE \\ \hline 3\,F \\ -GH \\ \hline 0 \\ \hline \end{array}$$

Sol. In the quotient, the first number is 5 and we know that 9 × 5 = 45.

:. D = 4 and E = 5.

Now, 48 – 45 = 3. Therefore, A = 8.

Also, to make the number 3F to be divisible by 9 we must have F = 6. And so, C = 4 and B = 6. Also, G = 3, H = 6.

Thus, the division works out as shown below :

$$\begin{array}{r} 5\,4 \\ 9\overline{)\,4\,8\,6} \\ -4\,5 \\ \hline 3\,6 \\ -3\,6 \\ \hline 0 \\ \hline \end{array}$$

MORE PUZZLES

Illustration 7.9

What number comes next :

1, 1, 2, 3, 5, 8, 13, 21, _,

Sol. We add the first two numbers, 1 + 1 and get the third number 2. To get 3, we add the two numbers that comes before it 1+ 2 = 3

Similarly, we go on doing that. So, 13 + 21 = 34 .

Illustration 7.10

Using digits 1, 2, 3.....9 (each exactly once). Can you find 3-digit numbers so that the second number is twice the first number and the third number is thrice the first number ?

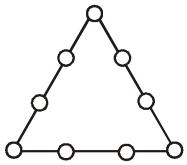
Sol. 192, 384, 576 (192 × 2 = 384, 192 × 3 = 576)

219, 438, 657 (219 × 2 = 438, 219 × 3 = 657)

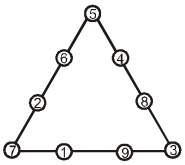
327, 654, 981 (327 × 2 = 654, 327 × 3 = 981)

Illustration 7.11

Put the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 in the bubbles so that each edge adds to the same things ?



Sol.



Make another one with different propositions of numbers.

Illustration 7.12

Insert '+' and '–' symbols between the numbers so that the equations become correct.

(i) 5 9 1 7 2 3 = 9 **(ii)** 1 2 3 4 5 8 = 7 **(iii)** 8 7 5 4 4 2 = 8 .

Sol. **(i)** 5 – 9 + 1 + 7 + 2 + 3 = 9 **(ii)** 1 + 2 – 3 + 4 – 5 + 8 = 7

(iii) 8 – 7 + 5 – 4 + 4 + 2 = 8

Illustration 7.13

Find two numbers whose product is a one-digit number and the sum is a two-digit number.

Sol. The two natural number are 1 and 9.
Their sum is 1 + 9 = 10 (i.e. a two digit number)
Their product is 1 × 9 = 9 (i.e. a single digit number).

Illustration 7.14

Find three whole numbers whose product and the sum are equal.

Sol. The three whole number are 1, 2 and 3.
Their sum is 1 + 2 + 3 = 6
Their product is 1 × 2 × 3 = 6.

Ask yourself

1. The sum of the digits of a 2-digit number is 7. If the number obtained by interchanying the digits is 27 more than the original number. Find the original number.

2. Replace * by the smallest digit so that 21 * 7 is divisible by
(a) 3 **(b)** 9

3. If a number is divisible by 3, need it be tested for 9? Justify your answer by stating any 2 numbers which are divisble by 3 but not by 9?

Answers

1. 52 **2.** **(a)** 2 **(b)** 8

Add your knowledge

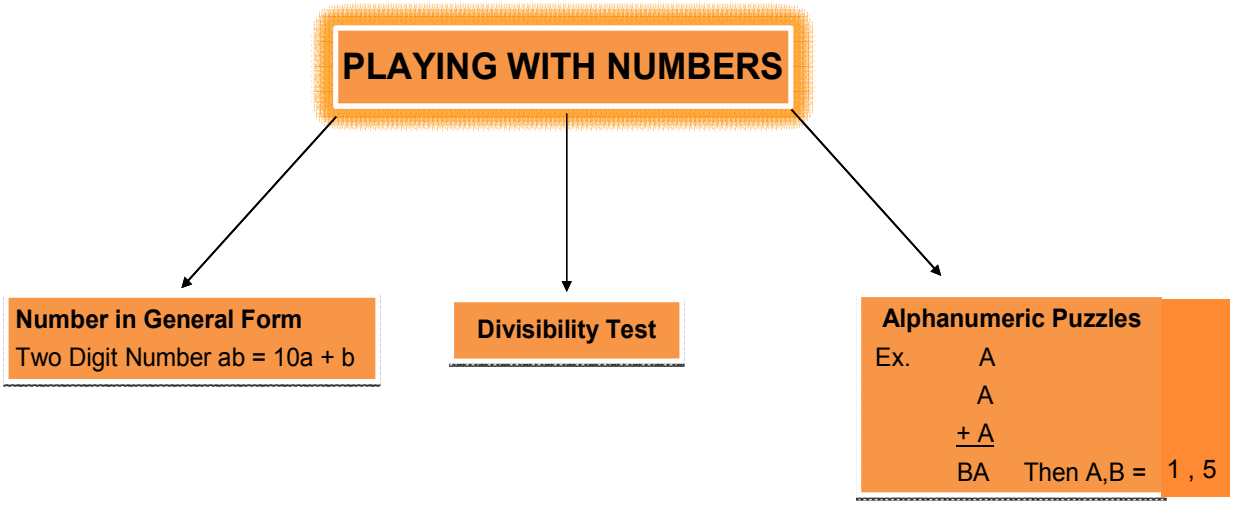
1. Divisiblility rule of '3'

Proof: Consider a 3 digit number 'abc'
So, 'abc' = 100a + 10b + c
Which can be written as = 99a + 9b + a + b + c
'abc' = 3(33a + 3b) + (a + b + c)
The first part is always divisible by '3'.
For 'abc' to be divisible by '3', the second part i.e., (a + b + c) must be divisible by 3.

2. Divisibility rule of '11'.

Proof : Consider a four digit number 'abcd'
'abcd' = 1000 a + 100 b + 10 c + d
On regrouping
'abcd' = (1001a – a) + (99b + b) + (11c – c) + d
'abcd' = (1001 a + 99b + 11 c) + (– a + b – c + d)
First part is always divisible by '11'
For the number 'abcd' to be divisible by '11', second part i.e., (b + d) – (a + c) must be zero or divisible by '11'.

Concept Map



Summary

1. In general, any two digit number xy made of digits x and y can be written as xy = 10 × x + y.
2. The sum of a two digit number and its reversing number is always a multiple of 11.
3. The difference of a two digit number and its reversing number is always a multiple of 9.

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

1. If a 3-digit number 'abc' is divisible by 11, then _____.
(A) a + b + c is a multiple of 11 (B) a + b – c is a multiple of 11
(C) a – b + c is a multiple of 11 (D) a – b – c is a multiple of 11

2. The number (10^n-1) is divisible by 11 for
(A) $n \in N$ (B) Odd values of n
(C) Even values of n (D) n is the multiple of 11

3. If the eight digit number 2575d568 is divisible by 54, the value of the digit 'd' is
(A) 4 (B) 7 (C) 0 (D) 8

4. 237 * 5 is divisible by 15. Then * = ?
(A) 1 (B) 4 (C) 7 (D) all of these

5. 635 * 2 is divisible by 6. Then the value of * = ?
(A) 2 (B) 3 (C) 6 (D) 1

6. If $\begin{array}{r} 1\ A\ B \\ +\ C\ C\ A \\ \hline 6\ 9\ 7 \end{array}$ and there is no carry on addition, then the value of B is _____.
(A) 5 (B) 4 (C) 3 (D) 2

7. If $\begin{array}{r} 1\ A \\ \times\ A \\ \hline B6 \end{array}$, where A and B are single digit number such that B – A = 3, then the values of A and B are
(A) 4, 5 (B) 9, 6 (C) 5, 4 (D) 6, 9

8. What is the value of A if each letter represents a different digit ?
$\begin{array}{r} A3B \\ \times\ B \\ \hline 217B \end{array}$
(A) 3 (B) 4 (C) 5 (D) 7

Directions : (9 to 10) In the following questions some/letters stands for arithmetic sign as indicated below. The remaining letters have their serial numbers in the Alphabets. Decode the letters into number and sign to decide correct alternative-
A = ×, E = –, O = ÷, U = +

9. TEF
(A) 14 (B) 12 (C) 16 (D) 18

10. SETUH
(A) 5 (B) 6 (C) 7 (D) 8

FILL IN THE BLANKS

1. 1234567?7654321 is divisible by 11, then ?

2. 5,8,13,21,34,_____

3. 7, 21, 84, 420, _____

Use +, –, × to fill the blanks

4. 18 ÷ 6 __ 5 + 2 __ 4 = 6

5. 8 __ 7 __ 3 =12

6. 8 ÷ 4 __ 2 __ 3 = 7

TRUE / FALSE

1. 13650275 is divisible by 25

2. 1289739 is divisible by 11.

3. 1004001 is divisible by 3 but not by 9

4. Every number divisible by 2 is also divisible by 4.

5. Every number divisible by 9 is also divisible by 3

MATCH THE COLUMN

1.

Column–I		**Column–II**	
(A)	abc – cba	(p)	divisible by 2
(B)	ab5	(q)	divisible by 111
(C)	ab8	(r)	divisible by 99
(D)	abc + bca + cab	(s)	divisible by 5

2.

Column –I		**Column–II**	
(A)	The sum of a number and the number obtained by reversing its digits is always divisible by	(p)	5
(B)	If the sum of the digits of a two- digit number is subtracted from the number then the resultant number is divisible by	(q)	2
(C)	If a is divisible by 5 and b is divisible by 5 then a+b is divisible by	(r)	9
(D)	If a is divisible by 4 and b is divisible by 4 then a–b is divisible by	(s)	11

SECTION -B (FREE RESPONSE TYPE)

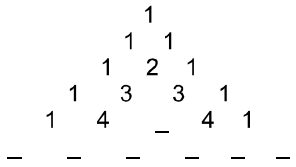
VERY SHORT ANSWER TYPE

1. What least value must be assigned to * so that the number 197 * 5462 is divisible by 9 ?

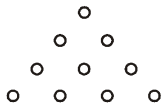
2. Find the number of pairs of x, y for which the number 357y25x is divisible by both 3 and 5.

3. Solve : (x – a) (x – b) (x – c).....(x – z) = ?

4. What numbers goes in the blank spaces ?



5. Shifting only 3 circles, make this shape upside down ?



SHORT ANSWER TYPE

6. A boy was asked to multiply a certain number by 25. He multiplied it by 52 and got his answer more than the correct one by 324. Find the number to be multiplied.

7. A number of two digits has 3 for its units digit and the sum of digits is $\frac{1}{7}$ of the number itself. What is the number ?

8. Find the value of $\left(1+\frac{1}{2}\right)\ \left(1+\frac{1}{3}\right)\ \left(1+\frac{1}{4}\right)\ \left(1+\frac{1}{5}\right)\ \left(1+\frac{1}{6}\right)\ \left(1+\frac{1}{7}\right)\ \left(1+\frac{1}{8}\right)$

9. Find the value of : $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ \ldots\ldots\ldots\ldots\ \left(1-\frac{1}{n}\right)$

10. You just bought twelve articles (small trees). You want to put them in six lines with four trees in each lines. How are you going to do it ?

LONG ANSWER TYPE

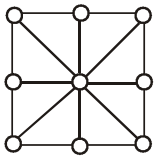
11. A two-digit number becomes five-sixth of itself when its digits are reversed. The two digits, differ by one. What is the number ?

12. The digit in the ten's place of a two-digit number is three times that in the one's place. If the digits are reversed the new number will be 36 less than the original number. Find the number.

13.
$$\begin{array}{r} A\ \ B \\ \times\ \ \ 3 \\ \hline C\ A\ B \end{array}$$
, find the value of A + B + C.

14. Every minute, each amoeba splits into two amoebas. At 5 P.M., you put one amoeba in a box. At 6 P. M., the box is full of amoebas. When was the box half full ?

15. Put the numbers (1 to 9) in the bubbles so that each row, column & diagonal adds up to the same thing



Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

1. The number which is exactly divisible by 99 is _____.
(A) 3572404 (B) 135792 (C) 913464 (D) 114345

2. The least value must be given to x so that the number 91876 × 2 is divisible by 8 is _____.
(A) 1 (B) 2 (C) 3 (D) 4

3. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is _____.
(A) 16 (B) 24 (C) 48 (D) 96

4. If N divided by 5 leaves a remainder of 3, then one's digit of N must be _____ .
(A) 3 (B) 4 (C) 9 (D) 7

5. 15287 is divisible by _____ .
(A) 3 (B) 7 (C) 9 (D) None of thes

6. a + b + c + d = d + e + f + g = g + h + i = 17. Each of the alphabets denotes distinct natural numbers from 1 to 9. Find value of d + g.
(A) 8 (B) 6 (C) 11 (D) 10

Directions : (7 to 9) In the following questions find out the digits corresponding to the letters representing those digits in the multiplication give below.

$$\begin{array}{r} 9\,b\,c \\ \times\quad 3\,5\,d \\ \hline 3\,a\,4\,e \\ 4\,a\,3\,5 \\ 2\,9\,6\,1 \\ \hline 3\,4\,a\,3\,9\,e \end{array}$$

7. b stands for :
(A) 6 (B) 7 (C) 8 (D) 9

8. c stands for :
(A) 7 (B) 6 (C) 5 (D) 4

9. d stands for :
(A) 2 (B) 3 (C) 4 (D) 5

10. A number divided by 323 leaves 125 as remainder. What would be the remainder, if this number is divided by 19 ?
(A) 10 (B) 11 (C) 12 (D) 13

SECTION -B (TECHIE STUFF)

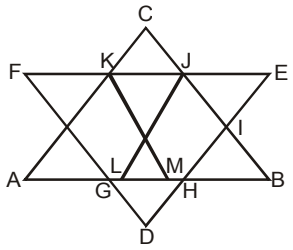
11. The least value of x so that 97215 × 6 is divisible by 11 is _____.
(A) 1 (B) 2 (C) 3 (D) 5

12. If 6 x y 5 is a four digit number divisible by 55, then (y – x) is equal to :
(A) – 2 (B) – 1 (C) 1 (D) 2

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

1. How many triangles are there in this figure 4 ? **[Aryabhatta-2004]**

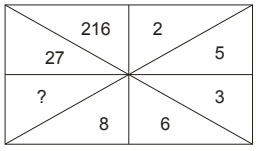


(A) 12 (B) 9 (C) 11 (D) 10

2. If '÷' means '–', '–' means 'x', 'x' means '+' and '+' means '÷' , then 20 × 60 ÷ 40 – 20 + 10 = **[Aryabhatta-2010]**
(A) 40 (B) 80 (C) 0 (D) 60

3. Which of the following numbers is exactly divisible by 99 ? **(IMO 2010)**
(A) 114345 (B) 135792 (C) 3572404 (D) 913464

4. Observe the pattern and find the number to fill the blank **[Aryabhatta-2011]**



(A) 25 (B) 125 (C) 215 (D) 625

5. The sum of a two digit number and the number obtained by interchanging the digits of the number is 121. If the digits of the number differ by 5, then find the number **[Aryabhatta-2011]**

6. A merchant has 140 litres, 260 litres and 320 litres of three kinds of oil. He wants to sell the oil by filling the three kinds of oil separately in tins of equal volumes. The volume of such a tin is_____. **(IMO 2011)**
(A) 20 litres (B) 13 litres (C) 16 litres (D) 70 litres

7. Find the next number in the series : **[Aryabhatta-2012]**
14, 28, 20, 40, 32, 64,.....
(A) 52 (B) 56 (C) 96 (D) 128

8. How many numbers are there between 200 and 800 which are divisible by both 5 and 7? **(IMO 2012)**

(A) 35 (B) 16 (C) 17 (D) Can't be determined

9. In the morning batch at 'a school' we have observed that when five students took seat on each bench, 4 students remained unseated. But when eleven students took seat per bench, 4 benches remained vacant. The number of students in the morning batch were? **(IMO 2012)**

(A) 55 (B) 48 (C) 26 (D) 44

10. Aisha needs to buy 300 sheets of construction paper. The office supply store sells construction paper in the following packages. Which of the following is the least expensive way for Aisha to buy 300 sheets of construction paper ? **(IMO 2012)**

Paper Purchase

Package	Number of Sheets	Price
W	50	₹ 4.50
X	75	₹ 5.10
Y	100	₹10.75
Z	150	₹12.25

(A) 6 packages of paper W (B) 4 packages of paper X
(C) 3 packages of paper Y (D) 2 packages of paper Z

11. Pratik had Rs.425.82 in his account. How much does he have in his account after he makes a deposit of Rs.120.75 and a withdrawal of Rs.185.90 ? **(IMO 2012)**

(A) Rs.12682 (B) Rs.538.82 (C) Rs.630.76 (D) Rs.360.67

12. Which of the following numbers is divisible by 11 ?

(A) 1011011 (B) 1111111 (C) 22222222 (D) 3333333

13. Three different containers contain different qualities of mixtures of milk and water, whose measurements are 403 kg. 434 kg and 465 kg. What biggest measure must be there to measure all the different quantities exactly ? **(IMO 2013)**

(A) 1 kg (B) 7 kg (C) 31 kg (D) 41 kg

14. If P and Q represents the prime digits, then find the value of P and Q respectively. **[Aryabhatta-2014]**

```
        P   P   5
      ×     Q   Q
  ---------------------
        2   Q   2   5
+   2 Q     2   5   0
  ---------------------
    2 5     5   7   5
  ---------------------
```

(A) 3, 3 (B) 7, 5 (C) 1, 5 (D) 7, 3

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	B	D	A	C	D	B	A	C

FILL IN THE BLANKS

1. 8 **2.** 55 **3.** 2520 **4.** –, × **5.** +,–

6. ×, +

TRUE / FALSE

1. True **2.** True **3.** True **4.** False **5.** True

MATCH THE COLUMN

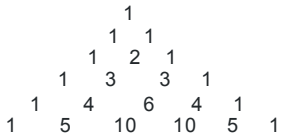
1. (A) – (r) ; (B) – (s); (C) – (p); (D) – (q)

2. (A) – (s) ; (B) – (r); (C) – (p); (D) – (q)

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

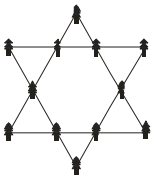
1. 2 **2.** 7 **3.** 0 **4.**



SHORT ANSWER TYPE

6. 12 **7.** 63 **8.** 4.5 **9.** $\frac{2}{n}$

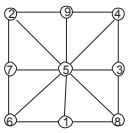
10.



LONG ANSWER TYPE

11. 54 **12.** 62 **13.** 6 **14.** 5 : 59 P. M.

15.



Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	D	A	C	A	D	B	C	A	C	B	C	B

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	C	A	B	83/38	A	B	C	D	B	D	C	C	D