MATHEMATICS

Class-IX

Topic-2

POLYNOMIALS

INDEX

CH-02 POLYNOMIALS

A. INTRODUCTION AND CLASSIFICATION OF POLYNOMIALS

(a) General Terms

(i) Constant : A symbol having a fixed numerical value is called a 'constant'
e.g. '2' has a definite value. So, it is a constant

(ii) Variable : A symbol which takes on various numerical values is called a 'variable'.
e.g. x,y ....

(iii) Coefficient : In the product of a constant and a variable, each is called the coefficient of the other.
e.g. In 6x, 6 is the coefficient of x.

(iv) Algebraic expression : Combination of constants and variables with (+), (–), (×), () is called an 'Algebraic expression'.
e.g. $17 - x$, $3x^2 - 4x + 12$, etc.

(v) Equation : Two expressions combined with equality symbol (=) is called an equation
e.g. $17 - x = 0$, $3x^2 - 4x + 12 = 2x^2 - 3x$. etc.

(vi) Degree of an expression : The highest number of times the variable is present in the terms of an expression is the degree of an expression.

(b) Types of Polynomial

An algebraic expression f(x) of the form $\mathbf{f(x) = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_nx^n}$, where $\mathbf{a_0, a_1, a_2, \dots\dots\dots, a_n}$ are real numbers and all the index of **'x'** are non-negative integers is called a **polynomial** in x.

• Identification of Polynomial :

For this, we have following examples :

(i) $\sqrt{3}x^2 + x - 5$ is a polynomial in variable x as all the exponents of x are non negative integers.

(ii) $\sqrt{3}x^2 + \sqrt{x} - 5x$ is not a polynomial as the exponent of second term $(\sqrt{x} = x^{1/2})$ is not a non negative integer.

(iii) $5x^3 + 2x^2 + 3x - \frac{5}{x} + 6$ is not a polynomial as the exponent of fourth term $-\frac{5}{x}$ is not non–negative integer.

• Degree of the Polynomial :

Highest index of **x** in algebraic expression is called the **degree of the polynomial**, here $a_0, a_1x, a_2x^2, \dots\dots\dots, a_nx^n$, are called the terms of the polynomial and $a_0, a_1, a_2, \dots\dots\dots, a_n$ are called various coefficients of the polynomial f(x).

For example:

(i) $p(x) = 3x^4 - 5x^2 + 2$ is a polynomial of degree 4

(ii) $q(x) = 5x^4 + 2x^5 - 6x^6 - 5$ is a polynomial of degree 6

(iii) $f(x) = 2x^3 + 7x - 5$ is a polynomial of degree 3.

• Different Types of Polynomials :

Generally, we divide the polynomials in the following categories.

(i) Based on degrees : There are four types of polynomials based on degrees. These are listed below :

→ **Zero degree polynomial :** Any non-zero number (constant) is regarded as a polynomial of degree zero or **zero degree polynomial**. i.e. f(x) = a, where a ≠ 0 is a zero degree polynomial, since we can write f(x) = a as $f(x) = ax^o$.

→ **Linear Polynomial :** A polynomial of degree one is called a **linear polynomial**. The general form of linear polynomial is **ax + b**, where **a** and **b** are any real constant and **a ≠ 0**.

→ **Quadratic Polynomial :** A polynomial of degree two is called a **quadratic polynomial**. The general form of a quadratic polynomial is $\mathbf{ax^2 + bx + c}$, where **a ≠ 0**.

→ **Cubic Polynomial :** A polynomial of degree three is called a **cubic polynomial**. The general form of a cubic polynomial is $\mathbf{ax^3 + bx^2 + cx + d}$, where **a ≠ 0**.

Biquadratic (or quartic) Polynomials : A polynomial of degree four is called a **biquadratic (quartic) polynomial**. The general form of a biquadratic polynomial is $\mathbf{ax^4 + bx^3 + cx^2 + dx + e}$, where **a ≠ 0**.

NOTE : A polynomial of degree five or more than five does not have any particular name. Such a polynomial usually called a polynomial of degree five or six or .... etc.

(ii) Based on number of terms

There are three types of polynomials based on number of terms. These are as follows :

→ **Monomial :** A polynomial is said to be a **monomial** if it has only one term. e.g. x, $9x^2$, $5x^3$ all are monomials.

→ **Binomial :** A polynomial is said to be a **binomial** if it contains two terms. e.g. $2x^2 + 3x$, $\frac{x}{2} + 5x^3$, $-8x^3 + 3$, all are binomials.

→ **Trinomials :** A polynomial is said to be a **trinomial** if it contains three terms. e.g. $3x^3 - 8x + \frac{5}{2}$, $5 - 7x + 8x^9$, $\sqrt{7}\,x^{10} + 8x^4 - 3x^2$ are all trinomials.

NOTE : A polynomial having four or more than four terms does not have particular name. These are simply called **polynomials**.

(c) Operation on polynomials

(i) Arithmetic operations over polynomials

(I) Addition : Addition of all like terms in given polynomials gives the sum of polynomials.

(II) Subtraction : The difference between the like term in given polynomials is known as subtraction of the given polynomials.

(III) Multiplication : multiply each term of the multiplicand by each term of the multiplier and take the algebraic sum of the products. This gives the product of the given polynomials.

(d) Division algorithm for polynomial

If p(x) and g(x) are any two polynomials with g(x) ≠ 0, then we can find polynomials r(x) and q(x) such that p(x) = g(x) × q(x) + r(x)
i.e. Dividend = (Divisor x Quotient) + Remainder
where r(x)=0 or degree of r(x) < degree of g(x).

(i) If r(x) = 0, g(x) is a factor of p(x)

(ii) If deg(p(x)) > deg(g(x)), then deg(q(x)) = deg(p(x)) –deg(g(x))

(iii) If deg(p(x)) = deg(g(x)), then deg(q(x)) = 0 and deg(r(x)) < deg(g(x))

(i) Value of a Polynomial :

The value of a polynomial **f(x)** at **x =** α is obtained by substituting x = α in the given polynomial and is denoted by **f(α).**

Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$,
If we replace x by – 2 everywhere in f(x), we get

$f(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6$

$f(-2) = -8 - 24 - 22 - 6$

$f(-2) = -60$.

So, we can say that value of f(x) at x = – 2 is – 60.

(ii) Zero or Root of a Polynomial :

The real number α is a root or zero of a polynomial **f(x)**, if **f(α) = 0**.
Consider the polynomial $f(x) = 2x^3 + x^2 - 7x - 6$,
If we replace x by 2 everywhere in f(x), we get

$f(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 - 6 = 0$

Hence, x = 2 is a root of f(x).

(iii) Remainder Theorem:

Let **'p(x)'** be any polynomial of degree greater than or equal to one and **a** be any real number and If **p(x)** is divided by **(x – a**), then the remainder is equal to **p(a)**.

(iv) Factor Theorem:

Let **p(x)** be a polynomial of degree greater than or equal to 1 and **'a'** be a real number such that **p(a) = 0**, then **(x – a)** is a factor of **p(x)**. Conversely, if **(x – a)** is a factor of **p(x),** then **p(a) = 0**.

Solved Examples

Example.1

Find the sum of the following : $P(x) = 4t^3 - 3t^2 + 2$, $Q(x) = t^4 - 2t^3 + 6$ and $R(x) = t^3 + 4t^2 - 4$

Sol. $P(x) = 4t^3 - 3t^2 + 2$

$Q(x) = t^4 - 2t^3 + 6$

$R(x) = t^3 + 4t^2 - 4$

$P(x) + Q(x) + R(x) = t^4 + 3t^3 + t^2 + 4$

Example.2

Subtract g(x) from f(x) where $f(x) = 2 + x^2 + 4x^3$, $g(x) = x^4 + x^2 + 3x + 5$.

Sol. $f(x) = 4x^3 + x^2 + 0.x + 2 = 0.x^4 + 4x^3 + x^2 + 0.x + 2$

$g(x) = x^4 + 0.x^3 + x^2 + 3x + 5$

$f(x) - g(x) = (0.x^4 + 4x^3 + x^2 + 0.x + 2) - (x^4 + 0x^3 + x^2 + 3x + 5)$

$f(x) - g(x) = (0 - 1)x^4 + (4 - 0)x^3 + (1 - 1)x^2 + (0 - 3)x + (2 - 5)$

$= -x^4 + 4x^3 + 0.x^2 - 3x - 3 = -x^4 + 4x^3 - 3x - 3.$

Example.3

Multiply : $(x^2 - 5x + 2)$ by $(3x^2 + 2x - 5)$

Sol. We have

$$\begin{array}{r} x^2 - 5x + 2 \\ \times \quad 3x^2 + 2x - 5 \\ \hline 3x^4 - 15x^3 + 6x^2 \qquad\qquad \\ + 2x^3 - 10x^2 + 4x \qquad \\ - 5x^2 - 25x + 10 \\ \hline 3x^4 - 13x^3 - 9x^2 + 29x - 10 \\ \hline \end{array}$$

Example.4

If $p(x) = x^2 - 2x + 1$ and $q(x) = x^3 - 3x^2 + 2x - 1$. Find $p(x) \times q(x)$ and check the degree of $p(x) \times q(x)$

Sol. $p(x) \times q(x) = (x^2 - 2x + 1) \times (x^3 - 3x^2 + 2x - 1)$

$= x^2(x^3 - 3x^2 + 2x - 1) - 2x(x^3 - 3x^2 + 2x - 1) + 1\,(x^3 - 3x^2 + 2x - 1)$

$= (x^5 - 3x^4 + 2x^4 + 2x^3 + 6x^3 + x^3 - x^2 - 4x^2 - 3x^2 + 2x + 2x - 1$

$= x^5 - 5x^4 + 9x^3 - 8x^2 + 4x - 1$

The degree of $p(x) \times q(x)$ is '5'

Example.5

What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol. Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

By long division method

$$\begin{array}{r|l} & x-2 \\ 3x^2+7x-6 & 3x^3+x^2-22x+9 \\ - & 3x^3 \;\underset{-}{+}7x^2 \underset{+}{-}6x \\ \hline & -6x^2+16x+9 \\ & \underset{+}{-}6x^2 \underset{+}{-}14x \underset{-}{+}12 \\ \hline & -2x-3 \end{array}$$

Hence if in p(x) we added 2x + 3 then it is exactly divisible by $3x^2 + 7x - 6$.

Example.6

What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$.

Sol: Let $p(x) = x^3 - 6x^2 - 15x + 80$ so that it is exactly divisible by $q(x) = x^2 + x - 12$.

We know if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

By long division method

$$\begin{array}{r|l} & x-7 \\ x^2+x-12 & x^3-6x^2-15x+80 \\ & \underset{-}{}x^3 \;\underset{-}{+}x^2 \underset{+}{-}12x \\ \hline & -7x^2-3x+80 \\ & \underset{+}{-}7x^2 \underset{+}{-}7x \underset{-}{+}84 \\ \hline & 4x-4 \end{array}$$

Hence, if in p(x) we subtract 4x – 4 then it is exactly divisible by $x^2 + x - 12$.

Example.7

If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$\Rightarrow \quad f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$

$\Rightarrow \quad 6 \times \frac{64}{9 \times 3} - 11 \times \frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow \quad 128 - 176 + 12k - 180 = 0$

$\Rightarrow \quad 12k + 128 - 356 = 0$

$\Rightarrow \quad 12k = 228$

$\Rightarrow \quad k = 19.$

Example.8

If x = 2 & x = 0 are two roots of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$. Find the values of a and b.

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 16 - 20 + 2a + b = 0$

$\Rightarrow \quad 2a + b = 4 \qquad ...(i)$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow \quad b = 0$

Put b = 0 in eq. (i)

$\Rightarrow \quad 2a + 0 = 4$

So, $2a = 4$

$\Rightarrow \quad a = 2.$

Hence, a = 2, b = 0.

Example. 9

Find the remainder, when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $f(x) = x^3 - 6x^2 + 2x - 4$

Let, $1 - 2x = 0$

$2x = 1$

$x = \frac{1}{2}$

Remainder = $f\left(\frac{1}{2}\right)$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - \frac{3}{2} + 1 - 4$$

$$= \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}.$$

Example.10

The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by x + 2 and if the remainder in each case is the same, find the value of a.

Sol. $p(x) = ax^3 + 3x^2 - 13$ and $q(x) = 2x^3 - 5x + a$

When p(x) & q(x) are divided by x + 2

Let $\quad x + 2 = 0 \qquad x = -2.$

$\therefore$ Remainder are same.

So, $p(-2) = q(-2)$

$\Rightarrow a(-2)^3 + 3(-2)^2 - 13 = 2(-2)^3 - 5(-2) + a$

$\Rightarrow -8a + 12 - 13 = -16 + 10 + a$

$\Rightarrow -9a = -5 \qquad \Rightarrow \qquad a = \frac{5}{9}.$

Example.11

If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by x – 1 and x + 1, the remainders are respectively 5 and 19. Determine the remainder when f(x) is divided by (x – 2).

Sol. When f(x) is divided by (x – 1) and (x + 1) the remainders are 5 and 19 respectively.

$\therefore \quad f(1) = 5$

$\Rightarrow \quad 1^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$\Rightarrow \quad 1 - 2 + 3 - a + b = 5$

$\Rightarrow \quad -a + b = 3 \qquad ...(i)$

and $\quad f(-1) = 19$

$\Rightarrow \quad (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$\Rightarrow \quad 1 + 2 + 3 + a + b = 19$

$\Rightarrow \quad a + b = 13 \quad \quad ...(ii)$

From equation (i) and (ii)

We have a = 5 and b = 8

So, $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

The remainder when f(x) is dividing by (x – 2) is equal to f(2).

$f(2) = 2^4 - 2(2^3) + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8$

$= 10.$

Example.12

The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by (x – 4) leaves remainder R_1 & R_2 respectively then find the value of 'a' if $2R_1 - R_2 = 0$.

Sol. Let $f(x) = ax^3 + 3x^2 - 3$ and $g(x) = 2x^3 - 5x + a$

$R_1 = f(4) = a(4)^3 + 3(4)^2 - 3$

$R_1 = 64a + 45.$

$R_2 = g(4) = 2(4)^3 - 5(4) + a$

$= 128 - 20 + a$

$= 108 + a.$

Given : $2R_1 - R_2 = 0$

$2(64a + 45) - (108 + a) = 0$

$128a + 90 - 108 - a = 0$

$127a = 18$

$a = \frac{18}{127}.$

Example.13

Show that x + 1 and 2x – 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x – 3) are factors of $2x^3 - 9x^2 + x + 12$ it is sufficient to show that p(–1) and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 - 1 + 12 = -12 + 12 = 0.$

And, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12 = \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12$

$= \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0.$

Hence, (x + 1) and (2x – 3) are the factors $2x^3 - 9x^2 + x + 12$.

Example.14

Find the values of a and b so that the polynomials $x^3 - ax^2 - 13x + b$ has (x – 1) and (x + 3) as factors.

Sol. Let $f(x) = x^3 - ax^2 - 13x + b$

Because (x – 1) and (x + 3) are the factors of f(x),

$\therefore f(1) = 0$ and $f(-3) = 0$

$f(1) = 0$

$\Rightarrow (1)^3 - a(1)^2 - 13(1) + b = 0$

$\Rightarrow 1 - a - 13 + b = 0$

$\Rightarrow -a + b = 12 \quad \quad .... (i)$

$f(-3) = 0$

$\Rightarrow (-3)^3 - a(-3)^2 - 13(-3) + b = 0$

$\Rightarrow -27 - 9a + 39 + b = 0$

$\Rightarrow$ $-9a + b = -12$ ....(ii)

Subtracting equation (ii) from equation (i)

$(-a + b) - (-9a + b) = 12 + 12$

$\Rightarrow$ $-a + 9a = 24$

$\Rightarrow$ $8a = 24$

$\Rightarrow$ $a = 3.$

Put a = 3 in equation (i)

$-3 + b = 12$

$\Rightarrow$ $b = 15.$

Hence, a = 3 and b = 15.

Example.15

If $ax^3 + bx^2 + x - 6$ has $x + 2$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, find the values of a and b?

Sol. Let $p(x) = ax^3 + bx^2 + x - 6$ be the given polynomial.

Now, $(x + 2)$ is a factor of p(x).

$p(-2) = 0$

$a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

$-8a + 4b - 2 - 6 = 0$

$-8a + 4b = 8$ .... (i)

It is given that p(x) leaves remainder 4 when it is divided by $(x - 2)$.

$p(2) = 4$

$a(2)^3 + b(2)^2 + (2) - 6 = 4$

$8a + 4b + 2 - 6 = 4$

$8a + 4b = 8$ .... (ii)

Add equation (i) & (ii)

$-8a + 4b + 8a + 4b = 8 + 8$

$8b = 16$ $\quad$ $b = 2.$

Put b = 2 in equation (i)

$-8a + 4(2) = 8$

$-8a + 8 = 8$

$-8a = 0$ $\quad$ $a = 0.$

Hence, a = 0 and b = 2.

Check Your Level

1. Classify the following polynomials based on number of terms.

(a) $x + 3$ **(b)** $x^2 + x + 2$ **(c)** $x^3 + 1$ **(d)** $8x^3$

(e) $7x^2 + 8x + 3$ **(f)** $\frac{x^3}{12}$ **(g)** $x^4 + \frac{x^2}{2}$ **(h)** $x^2 + x + 3$

2. Classify the following polynomials based on their degree.

(a) $3x^2 + 4x$ **(b)** $2x^3 + \frac{x}{2} + 3$ **(c)** $7x + 2$ **(d)** $5x^2$

(e) $x^3 + \sqrt{2}x^2 + 1$ **(f)** $x^3 - 1$ **(g)** $8x + 3$

3. Find zeroes of the following polynomials

(a) $7x - 14$ **(b)** $8x + 1$ **(c)** $x^2 - 5x - 6$ **(d)** $2x^2 + 3x + 1$

4. Divide $6x^2 + 13x + 16$ by $2x + 3$ and find the quotient and remainder.

5. The polynomial $5x^2 + 7x + 3$ is divided by $x - 2$. Find the remainder by using remainder theorem.

6. Examine whether $(a - 1)$ is a factor of $a^3 - 3a^2 + 3a - 1$.

Answers

1.	**(a)**	binomial	**(b)**	trinomial	**(c)**	binomial	**(d)**	monomial
	(e)	trinomial	**(f)**	monomial	**(g)**	binomial	**(h)**	trinomial
2.	**(a)**	quadratic	**(b)**	cubic	**(c)**	linear	**(d)**	quadratic
	(e)	cubic	**(f)**	cubic	**(g)**	linear		
3.	**(a)**	x = 2	**(b)**	$x = -\frac{1}{8}$	**(c)**	x = –1 or 6	**(d)**	$x = -\frac{1}{2}$ or -1
4.	q = 3x + 2, r = 10		**5.**	37	**6.**	Yes it's a factor		

B. ALGEBRAIC IDENTITIES

Some important identities are :

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(v) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(vi) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(viii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case : if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

Value Form :

(i) $a^2 + b^2 = (a + b)^2 - 2ab$, if **a + b** and **ab** are given.

(ii) $a^2 + b^2 = (a - b)^2 + 2ab$, if **a – b** and **ab** are given.

(iii) $a + b = \sqrt{(a-b)^2 + 4ab}$, if **a – b** and **ab** are given.

(iv) $a - b = \sqrt{(a+b)^2 - 4ab}$, if **a + b** and **ab** are given.

(v) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$, if **a +** $\frac{1}{a}$ is given.

(vi) $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$, if **a –** $\frac{1}{a}$ is given.

(vii) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$, if **(a + b)** and **ab** are given.

(viii) $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$, if **(a – b)** and **ab** are given.

(ix) $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$, if **a +** $\frac{1}{a}$ is given.

(x) $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$, if **a –** $\frac{1}{a}$ is given.

(xi) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = [(a + b)^2 - 2ab](a + b)(a - b)$.

NOTE : (i) $(x^n - a^n)$ is divisible by $(x - a)$ for all the values of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ and $(x - a)$ for all the even values of n.

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all the odd values of n.

Solved Examples

Example.16

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}\left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}=1$$

Sol. $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}\left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

$= \left(x^{a-b}\right)^{a^2+ab+b^2}\left(x^{b-c}\right)^{b^2+bc+c^2}\left(x^{c-a}\right)^{c^2+ca+a^2} = \left(x^{a^3-b^3}\right)\left(x^{b^3-c^3}\right)\left(x^{c^3-a^3}\right)$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1.$

Example.17

Expand :

(i) $\left(2x-\frac{1}{3x}\right)^2$ **(ii)** $\left(3x^2+5y\right)^2$

(iii) $\left(\sqrt{2}x-3y\right)\left(\sqrt{2}x+3y\right)$ **(iv)** $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Sol. **(i)** $\left(2x-\frac{1}{3x}\right)^2 = (2x)^2 - 2(2x)\left(\frac{1}{3x}\right) + \frac{1}{(3x)^2} = 4x^2 - \frac{4}{3} + \frac{1}{9x^2}$.

(ii) $(3x^2 + 5y)^2 = (3x^2)^2 + 2(3x^2)(5y) + (5y)^2 = 9x^4 + 30x^2y + 25y^2$

(iii) $(\sqrt{2}x - 3y)(\sqrt{2}x + 3y) = (\sqrt{2}x)^2 - (3y)^2 = 2x^2 - 9y^2$

(iv) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{ab}{4} - b + \frac{a}{2}.$

Example.18

Simplify :

(i) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$ **(ii)** $(2x+y)(2x-y)(4x^2+y^2)$

(iii) $(x + y - 2z)^2 - x^2 - y^2 - 3z^2 + 4xy$

Sol. **(i)** $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right) = \left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left[(x^2)^2 - \left(\frac{1}{x^2}\right)^2\right]\left(x^4+\frac{1}{x^4}\right) = \left(x^4-\frac{1}{x^4}\right)\left(x^4+\frac{1}{x^4}\right)$

$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2$

$= x^8 - \frac{1}{x^8}.$

(ii) $(2x + y)(2x - y)(4x^2 + y^2) = [(2x)^2 - (y)^2](4x^2 + y^2)$

$= (4x^2 - y^2)(4x^2 + y^2) = (4x^2)^2 - (y^2)^2 = 16x^4 - y^4.$

(iii) $(x + y - 2z)^2 - x^2 - y^2 - 3z^2 + 4xy$

$= x^2 + y^2 + (-2z)^2 + 2(x)(y) + 2(y)(-2z) + 2(-2z)(x) - x^2 - y^2 - 3z^2 + 4xy$

$= x^2 + y^2 + 4z^2 + 2xy - 4yz - 4zx - x^2 - y^2 - 3z^2 + 4xy$

$= z^2 + 6xy - 4yz - 4zx.$

Example.19

Evaluate :

(i) $(107)^2$ **(ii)** $(94)^2$ **(iii)** $(0.99)^2$

Sol. **(i)** $(107)^2 = (100 + 7)^2$

$= (100)^2 + (7)^2 + 2 \times 100 \times 7$

$= 10000 + 49 + 1400$

$= 11449$

(ii) $(94)^2 = (100 - 6)^2$

$= (100)^2 + (6)^2 - 2 \times 100 \times 6$

$= 10000 + 36 - 1200$

$= 8836$

(iii) $(0.99)^2 = (1 - 0.01)^2$

$= (1)^2 + (0.01)^2 - 2 \times 1 \times 0.01$

$= 1 + 0.0001 - 0.02$

$= 0.9801$

Example.20

If $x^2 + \frac{1}{x^2} = 23$, find the values of $\left(x + \frac{1}{x}\right)$, $\left(x - \frac{1}{x}\right)$ and $\left(x^4 + \frac{1}{x^4}\right)$.

Sol. $x^2 + \frac{1}{x^2} = 23$ ...(i)

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$ [Adding 2 on both sides of (i)]

$\Rightarrow (x^2) + \left(\frac{1}{x}\right)^2 + 2 \cdot x \cdot \frac{1}{x} = 25$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (5)^2$

$\Rightarrow x + \frac{1}{x} = 5$

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 23 - 2 = 21$

$\Rightarrow \left(x - \frac{1}{x}\right) = \pm\sqrt{21}.$

$\left(x^2 + \frac{1}{x^2}\right)^2 = \left(x^4 + \frac{1}{x^4}\right) + 2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = (23)^2 - 2 = 529 - 2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 527.$

Example. 21

Prove that : $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

Sol. Here, L.H.S. $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right]$

$= \frac{1}{2}\left[(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)\right]$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$ = RHS **Hence Proved.**

Example.22

If a + b + c = 9 and ab + bc + ca = 23, then find the value of $a^2 + b^2 + c^2$.

Sol. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(9)^2 = a^2 + b^2 + c^2 + 2(23)$

$a^2 + b^2 + c^2 = 81 - 46$

$a^2 + b^2 + c^2 = 35$.

Example.23

Expand :

(i) $\left(\frac{1}{3x} - \frac{2}{5y}\right)^3$ **(ii)** $(4 + 3x)^3$

Sol. (i) $\left(\frac{1}{3x} - \frac{2}{5y}\right)^3$

$= \left(\frac{1}{3x}\right)^3 - \left(\frac{2}{5y}\right)^3 - 3\left(\frac{1}{3x}\right)\left(\frac{2}{5y}\right)\left(\frac{1}{3x} - \frac{2}{5y}\right)$

$= \frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{5xy}\left(\frac{1}{3x} - \frac{2}{5y}\right)$

$= \frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{15x^2y} + \frac{4}{25xy^2}$

(ii) $(4 + 3x)^3$

$= (4)^3 + (3x)^3 + 3(4)(3x)(4 + 3x)$

$= 64 + 27x^3 + 36x\,(4 + 3x)$

$= 64 + 27x^3 + 144x + 108x^2$

Example.24

Simplify :

(i) $(3x + 4)^3 - (3x - 4)^3$ **(ii)** $\left(x + \frac{2}{x}\right)^3 + \left(x - \frac{2}{x}\right)^3$

Sol. (i) $(3x + 4)^3 - (3x - 4)^3$

$= [(3x)^3 + (4)^3 + 3\,(3x)\,(4)\,(3x + 4)] - [(3x)^3 - (4)^3 - 3\,(3x)\,(4)\,(3x - 4)]$

$= [27x^3 + 64 + 36x\,(3x + 4)] - [27x^3 - 64 - 36x\,(3x - 4)]$

$= [27x^3 + 64 + 108x^2 + 144x] - [27x^3 - 64 - 108x^2 + 144x]$

$= 27x^3 + 64 + 108x^2 + 144x - 27x^3 + 64 + 108x^2 - 144x$

$= 128 + 216x^2$.

(ii) $\left(x + \frac{2}{x}\right)^3 + \left(x - \frac{2}{x}\right)^3$

$= x^3 + \left(\frac{2}{x}\right)^3 + 3(x)\left(\frac{2}{x}\right)\left(x+\frac{2}{x}\right) + x^3 - \left(\frac{2}{x}\right)^3 - 3(x)\left(\frac{2}{x}\right)\left(x-\frac{2}{x}\right)$

$= x^3 + \frac{8}{x^3} + 6x + \frac{12}{x} + x^3 - \frac{8}{x^3} - 6x + \frac{12}{x}$

$= 2x^3 + \frac{24}{x}$.

Example.25

Evaluate :

(i) $(1005)^3$ **(ii)** $(997)^3$

Sol. **(i)** $(1005)^3 = (1000 + 5)^3$

$= (1000)^3 + (5)^3 + 3\,(1000)\,(5)\,(1000 + 5)$

$= 1000000000 + 125 + 15000\,(1000 + 5)$

$= 1000000000 + 125 + 15000000 + 75000$

$= 1015075125$.

(ii) $(997)^3 = (1000 - 3)^3$

$= (1000)^3 - (3)^3 - 3 \times 1000 \times 3 \times (1000 - 3)$

$= 1000000000 - 27 - 9000 \times (1000 - 3)$

$= 1000000000 - 27 - 9000000 + 27000$

$= 991026973$.

Example.26

If $x - \frac{1}{x} = 5$, find the value of $x^3 - \frac{1}{x^3}$.

Sol. We have, $x - \frac{1}{x} = 5$ ...(i)

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = (5)^3$ [Cubing both sides of (i)]

$\Rightarrow x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \cdot \left(x - \frac{1}{x}\right) = 125$ $\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 125$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 5 = 125$ [Substituting $\left(x - \frac{1}{x}\right) = 5$]

$\Rightarrow x^3 - \frac{1}{x^3} - 15 = 125$ $\Rightarrow x^3 - \frac{1}{x^3} = (125 + 15) = 140$.

Example.27

Find the products of the following expression :

(i) $(4x + 3y)\,(16x^2 - 12xy + 9y^2)$

(ii) $(5x - 2y)\,(25x^2 + 10xy + 4y^2)$

Sol. **(i)** $(4x + 3y)\,(16x^2 - 12\,xy + 9y^2)$

$= (4x + 3y)\,[(4x)^2 - (4x) \times (3y) + (3y)^2]$

$= (a + b)\,(a^2 - ab + b^2)$ [Where $a = 4x$, $b = 3y$]

$= a^3 + b^3$

$= (4x)^3 + (3y)^3 = 64x^3 + 27y^3$.

(ii) $(5x - 2y)\,(25x^2 + 10xy + 4y^2)$

$= (5x - 2y)\,[(5x)^2 + (5x) \times (2y) + (2y)^2]$

$= (a - b)\,(a^2 + ab + b^2)$ [Where $a = 5x$, $b = 2y$]

$= a^3 - b^3$

$= (5x)^3 - (2y)^3$

$= 125x^3 - 8y^3$.

Example.28

Find the product of following expression :

(i) $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$

(ii) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

Sol. **(i)** $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$

Let, $a = 3x$, $b = -4y$, $c = 5z$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a^3 + b^3 + c^3 - 3abc)$

$= (3x)^3 + (-4y)^3 + (5z)^3 - 3(3x)(-4y)(5z)$

$= 27x^3 - 64y^3 + 125z^3 + 180\,xyz$

(ii) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

Let $x = 2a$, $y = -3b$, $z = -2c$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x^3 + y^3 + z^3 - 3xyz)$

$= (2a)^3 + (-3b)^3 + (-2c)^3 - 3(2a)(-3b)(-2c)$

$= 8a^3 - 27b^3 - 8c^3 - 36abc$

Example.29

If $a + b + c = 9$ and $ab + bc + ac = 26$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Sol. We have $a + b + c = 9$ ...(i)

$\Rightarrow$ $(a + b + c)^2 = 81$ [On squaring both sides of (i)]

$\Rightarrow$ $a^2 + b^2 + c^2 + 2(ab + bc + ac) = 81$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2 \times 26 = 81$ [ab + bc + ac = 26]

$\Rightarrow$ $a^2 + b^2 + c^2 = (81 - 52)$

$\Rightarrow$ $a^2 + b^2 + c^2 = 29.$

Now, we have

$a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$= (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ac)]$

$= 9 \times [(29 - 26)]$

$= (9 \times 3) = 27.$

Example.30

Simplify : $\dfrac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$.

Sol. Here, $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\therefore$ $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3\ (a^2 - b^2)\ (b^2 - c^2)\ (c^2 - a^2)$

Also, $(a - b) + (b - c) + (c - a) = 0$

$\therefore$ $(a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)\ (b - c)\ (c - a)$

$= \dfrac{3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{3(a - b)(b - c)(c - a)}$

$= \dfrac{3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)}{3(a - b)(b - c)(c - a)}$

$= (a + b)(b + c)(c + a)$.

Example.31

Prove that : $(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$.

Sol. Let $(x-y) = a$, $(y-z) = b$ and $(z-x) = c$.

Then, $a + b + c = (x-y) + (y-z) + (z-x) = 0$

$a^3 + b^3 + c^3 = 3abc$

or $(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$.

Example.32

Find the value of $(28)^3 - (78)^3 + (50)^3$.

Sol. Let $a = 28$, $b = -78$, $c = 50$

Then, $a + b + c = 28 - 78 + 50 = 0$

$a^3 + b^3 + c^3 = 3abc$.

So, $(28)^3 + (-78)^3 + (50)^3 = 3 \times 28 \times (-78) \times 50 = -327600$.

Check Your Level

1. Expand $(2x + 3y - 2z)^2$.

2. If $a + b = 7$ and $ab = 12$, find the value of $a^3 + b^3$.

3. If $a + \frac{1}{a} = 5$ then $a^2 + \frac{1}{a^2}$ is

4. If $a + \frac{1}{a} = 4$ then $a^3 + \frac{1}{a^3}$ is equal to

5. If $p - q = 9$, prove that $p^3 - q^3 - 27pq = 729$.

Answers

1. $4x^2 + 9y^2 + 4z^2 + 12xy - 12yz - 8xz$ **2.** 91 **3.** 23

4. 52

C. FACTORIZATION

To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called **factorization**.

(a) Factorization by taking out the common factor :

Working Rule: When each term of an expression has a common factor, divide each term by this factor and take out as a multiple.

(b) Factorization by grouping :

Working Rule:Sometimes in a given expression it is not possible to take out a common factor directly.However, the terms of the given expression are grouped in such a manner that we may have a common factor.This can be factorized as discussed above.

(c) Factorization by making a perfect square :

Working Rule : $a^2 + 2ab + b^2 = (a + b)^2$

(d) Factorization the difference of two squares :

Working Rule: $a^2 - b^2 = (a + b)(a - b)$

(e) Factorization of a Quadratic Polynomial by Splitting the Middle Term :

Working Rule:

Case 1: Polynomials of the form x^2+bx+c

we find integers p and q such that p+q=b and pq=c. Then,

$x^2 + bx + c = x^2 + (p + q)x + pq$

$= x^2 + px + qx + pq$

$= x(x + p) + q(x + p)$

$= (x + p)(x + q)$

Case 2: Polynomials of the form ax^2+bx+c

we find integers p and q such that p+q=b and pq=ac. Then,

$ax^2 + bx + c = ax^2 + (p + q)\,x + \frac{pq}{a}$

$= a^2x^2 + a(p + q)x + pq$

$= ax(ax + p) + q(ax + p)$

$= (ax + p)(ax + q)$

(f) Factorization of an algebraic expression as the sum or difference of two cubes

Working Rule: (i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(ii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(g) Factorization of an algebraic expression of the form $a^3 + b^3 + c^3 - 3abc$:

Working Rule: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case : if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

Solved Examples

Example.33

Factorize :

(i) $2a(x + y) - 3b(x + y)$ **(ii)** $x(x + y)^3 - 3x^2y(x + y)$

(iii) $8(3a - 2b)^2 - 10(3a - 2b)$

Sol. (i) $2a(x + y) - 3b(x + y)$

$= (x + y)(2a - 3b)$.

(ii) $x(x + y)^3 - 3x^2y(x + y)$

$= x(x + y)[(x + y)^2 - 3xy]$

$= x(x + y)[x^2 + y^2 + 2xy - 3xy]$

$= x(x + y)[x^2 + y^2 - xy]$

$= x(x^3 + y^3)$.

(iii) $8(3a - 2b)^2 - 10(3a - 2b)$

$= 2(3a - 2b)[4(3a - 2b) - 5]$

$= 2(3a - 2b)[12a - 8b - 5]$.

Example.34

Factorize :

(i) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$ **(ii)** $(x^2 + 3x)^2 - 5(x^2 + 3x) - y(x^2 + 3x) + 5y$

Sol. (i) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x} = \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$.

(ii) $(x^2 + 3x)^2 - 5(x^2 + 3x) - y(x^2 + 3x) + 5y$

$= (x^2 + 3x)(x^2 + 3x - 5) - y(x^2 + 3x - 5)$

$= (x^2 + 3x - 5)(x^2 + 3x - y)$.

Example.35

Factorize :

(i) $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$ **(ii)** $2a^2 + 2\sqrt{6}\,ab + 3b^2$

(iii) $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$

Sol. **(i)** $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$

Let, $x - y = a$ & $x + y = b$

$= 4a^2 - 12ab + 9b^2$

$= (2a)^2 - 2(2a)(3b) + (3b)^2$

$= (2a - 3b)^2$

$= [2(x-y) - 3(x+y)]^2$

$= [2x - 2y - 3x - 3y]^2$

$= [-x - 5y]^2$.

(ii) $2a^2 + 2\sqrt{6}\,ab + 3b^2$

$= \left(\sqrt{2}a\right)^2 + 2\left(\sqrt{2}a\right)\left(\sqrt{3}b\right) + \left(\sqrt{3}b\right)^2$

$= \left(\sqrt{2}a + \sqrt{3}b\right)^2$.

(iii) $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$

$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$

$= (5x - 2y + 3z)^2$.

Example.36

Factorize :

(i) $x^8 - y^8$ **(ii)** $x^4 + 5x^2 + 9$ **(iii)** $x^4 + 4x^2 + 3$ **(iv)** $x^4 + x^2y^2 + y^4$

Sol. **(i)** $x^8 - y^8$

$= (x^4)^2 - (y^4)^2$

$= (x^4 + y^4)(x^4 - y^4)$

$= (x^4 + y^4)[(x^2)^2 - (y^2)^2]$

$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$

$= (x^4 + y^4)(x^2 + y^2)(x - y)(x + y)$.

(ii) $x^4 + 5x^2 + 9$

$= (x^2)^2 + 5x^2 + (3)^2$

$= (x^2)^2 + 6x^2 + (3)^2 - x^2$

$= (x^2 + 3)^2 - (x)^2$

$= (x^2 + 3 + x)(x^2 + 3 - x)$.

(iii) $x^4 + 4x^2 + 3$

$= (x^2)^2 + 2(2)x^2 + (2)^2 - 1$

$= (x^2 + 2)^2 - (1)^2$

$= (x^2 + 2 + 1)(x^2 + 2 - 1)$

$= (x^2 + 3)(x^2 + 1)$.

(iv) $x^4 + x^2y^2 + y^4$

$= (x^2)^2 + 2.x^2.y^2 + (y^2)^2 - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$.

Example.37

Factorize :

(i) $x^2 + 6\sqrt{2}\,x + 10$ **(ii)** $5\sqrt{5}\,x^2 + 20x + 3\sqrt{5}$

(iii) $2x^2 - \frac{5}{6}x + \frac{1}{12}$ **(iv)** $7(x - 2y)^2 - 25(x - 2y) + 12$

Sol. **(i)** $x^2 + 6\sqrt{2}\,x + 10 = x^2 + 5\sqrt{2}\,x + \sqrt{2}\,x + 10$

$= x(x + 5\sqrt{2}) + \sqrt{2}(x + 5\sqrt{2}) = (x + 5\sqrt{2})(x + \sqrt{2})$

(ii) $5\sqrt{5}x^2 + 20x + 3\sqrt{5} = 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$

$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3) = (5x + \sqrt{5})(\sqrt{5}x + 3)$

$= \sqrt{5}(\sqrt{5}x + 1)(\sqrt{5}x + 3)$

(iii) $2x^2 - \frac{5}{6}x + \frac{1}{12} = \frac{24x^2 - 10x + 1}{12}$

$= \frac{1}{12}(24x^2 - 4x - 6x + 1) = \frac{1}{12}[4x(6x - 1) - 1(6x - 1)]$

$= \frac{1}{12}(6x - 1)(4x - 1).$

(iv) $7(x - 2y)^2 - 25(x - 2y) + 12$

Let, $x - 2y = a = 7a^2 - 25a + 12$

$= 7a^2 - 21a - 4a + 12 = 7a(a - 3) - 4(a - 3)$

$= (a - 3)(7a - 4)$

$= (x - 2y - 3)(7x - 14y - 4)$

Example. 38

What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2 - 12x$.

Sol. Volume of cuboid $= 3x^2 - 12x = 3x(x - 4)$

Possible dimensions are :

Length = 3 unit, Breadth = x unit and Height = $(x - 4)$ unit.

Example.39

Factorize :

(i) $27a^3 + 125b^3$ **(ii)** $(a - 2b)^3 - 512b^3$

(iii) $x^9 - y^9$ **(iv)** $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

Sol. **(i)** $27a^3 + 125b^3$

$= (3a)^3 + (5b)^3$

$= (3a + 5b)[(3a)^2 + (5b)^2 - (3a)(5b)]$

$= (3a + 5b)[9a^2 + 25b^2 - 15ab].$

(ii) $(a - 2b)^3 - 512b^3$

$= (a - 2b)^3 - (8b)^3$

$= (a - 2b - 8b)[(a - 2b)^2 + (8b)^2 + (a - 2b)(8b)]$

$= (a - 10b)[a^2 + 4b^2 - 4ab + 64b^2 + 8ab - 16b^2]$

$= (a - 10b)[a^2 + 52b^2 + 4ab]$

(iii) $x^9 - y^9$

$= (x^3)^3 - (y^3)^3$

$= (x^3 - y^3)[(x^3)^2 + x^3y^3 + (y^3)^2]$

$= (x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6]$

(iv) $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

$= a^3 - \frac{1}{a^3} - 2\left(a - \frac{1}{a}\right)$

$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2}\right) - 2\left(a - \frac{1}{a}\right)$

$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - 2\right)$

$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right).$

Example.40

Prove that : $\dfrac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{0.87\times0.87-0.87\times0.13+0.13\times0.13}$ = 1.

Sol. $\dfrac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{0.87\times0.87-0.87\times0.13+0.13\times0.13}$

Let 0.87 = a and 0.13 = b

$= \dfrac{a^3+b^3}{a^2-ab+b^2} = \dfrac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2}$ = a + b = 0.87 + 0.13 = 1.0

Example.41

Factorize :

(i) $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc$ **(ii)** $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

(iii) $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

Sol. **(i)** $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc = (\sqrt{2}a)^3+(2b)^3+(-3c)^3-3(\sqrt{2}a)(2b)(-3c)$

$= (\sqrt{2}a+2b-3c)\left[(\sqrt{2}a)^2+(2b)^2+(-3c)^2-(\sqrt{2}a)(2b)-(2b)(-3c)-(-3c)(\sqrt{2}a)\right]$

$=(\sqrt{2}a+2b-3c)\left[2a^2+4b^2+9c^2-2\sqrt{2}ab+6bc+3\sqrt{2}ac\right]$

(ii) $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Let a = 2x – 3y, b = 4z – 2x and c = 3y – 4z

So, $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3 = a^3+b^3+c^3$

Now, a + b + c = 2x – 3y + 4z – 2x + 3y – 4z = 0

So, $a^3+b^3+c^3 = 3abc = 3(2x-3y)(4z-2x)(3y-4z)$.

(iii) $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

Let x = p(q – r), y = q(r – p), z = r(p – q)

So, $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3 = x^3+y^3+z^3$

Now, x + y + z = p(q – r) + q(r – p) + r(p – q) = pq – pr + qr – qp + rp – rq = 0

So, $x^3+y^3+z^3 = 3xyz = 3p(q-r)\,q(r-p)\,r(p-q) = 3pqr(q-r)(r-p)(p-q)$.

Example.42

If p = 2 – a, then find the value of $a^3+6ap+p^3-8$.

Sol. p = 2 – a

a + p – 2 = 0

Now, $a^3+6ap+p^3-8 = a^3+p^3+(-2)^3-3(a)(p)(-2)$

$= (a+p-2)(a^2+p^2+4-ap+2p+2a) = (0)(a^2+p^2+4-ap+2p+2a) = 0$.

Check Your Level

1. Factorize: $x^2(xy+5)-2y^2(xy+5)$
2. Factorize: $3x^2+8x+4$
3. Factorize: 2a + 4b – ac – 2bc
4. Factorize: $27a^3-b^3+c^3+9abc$.
5. Factorize: $32x^3-4d^3$

Answers

1. $(xy+5)(x^2-2y^2)$ 2. $(x+2)(3x+2)$ 3. $(2-c)(a+2b)$

4. $(3a-b+c)(9a^2+b^2+c^2+3ab+bc-3ac)$ 5. $4(2x-d)(4x^2+2xd+d^2)$

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

1. What is the degree of polynomial 2^5 ?

2. What is the degree of the zero polynomial ?

3. If $p(x) = x^2 - 2\sqrt{2}x + 1$,then find the value of $p(2\sqrt{2})$

4. If $x^2 + kx + 6 = (x + 2)(x + 3)$ for all x, then find the value of k.

5. If $x^{51} + 51$ is divided by $x + 1$, find the remainder.

6. Find the coefficient of x in the expansion of $(x + 3)^3$.

7. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then find the value of b.

8. If $a + b + c = 0$, then find the value of $a^3 + b^3 + c^3$.

9. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a.

10. Factorise :
(i) $x^2 + 9x + 18$ **(ii)** $6x^2 + 7x - 3$
(iii) $2x^2 - 7x - 15$ **(iv)** $84 - 2r - 2r^2$

11. Using suitable identity, evaluate the following :
(i) 103^3 **(ii)** 101×102 **(iii)** 999^2

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

12. **(i)** Check whether p(x) is a multiple of g(x) or not, where
$p(x) = x^3 - x + 1$, $g(x) = 2 - 3x$
(ii) Check whether g(x) is a factor of p(x) or not, where
$p(x) = 8x^3 - 6x^2 - 4x + 3$, $g(x) = \frac{x}{3} - \frac{1}{4}$

13. Find the value of a, if $x - a$ is a factor of $x^3 - ax^2 + 2x + a - 1$.

14. **(i)** Without actually calculating the cubes, find the value of $48^3 - 30^3 - 18^3$.
(ii) Without finding the cubes, factorise $(x - y)^3 + (y - z)^3 + (z - x)^3$.

15. For the polynomial $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write
(i) the degree of the polynomial **(ii)** the coefficient of x^3
(iii) the coefficient of x^6 **(iv)** the constant term

16. Give an example of a polynomial, which is:
(i) monomial of degree 1 **(ii)** binomial of degree 20
(iii) trinomial of degree 2

17. If $p(x) = x^2 - 4x + 3$, evaluate : $p(2) - p(-1) + p\left(\frac{1}{2}\right)$.

18. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial : $x^4 + 1; x - 1$

19. Show that :
(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.
(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

20. Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: [04 MARK EACH]

21. Factorise :
(i) $a^3 - 8b^3 - 64c^3 - 24abc$ **(ii)** $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$.

22. Find the value of
(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$
(ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

23. If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.

24. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

25. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

26. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

27. Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS [05 MARK EACH]

28. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

29. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the value of a. Also find the remainder when p(x) is divided by $x + 2$.

30. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction and classification of polynomials

A.1 What is the degree of the polynomial $\sqrt{5}$.

A.2 Find the coefficient of x in the expansion of $(x-4)^2$

A.3 A zero polynomial has how many zeroes?

A.4 If $x - 3$ is the factor of $ax^2 + 5x + 12$ find the value of a.

A.5 Determine whether $x - 3$ is a factor of polynomial $p(x) = x^3 - 3x^2 + 4x - 12$.

A.6 Using factor theorem, prove that p(x) is divisible by g(x) if $P(x) = 4x^4 + 5x^3 - 12x^2 - 11x + 5$, $g(x) = 4x + 5$.

A.7 If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by $x - 2$, find the value of a.

A.8 Find the value of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $x^2 - 1$.

Section (B) : Algebraic identity

B.1 Evaluate : $(999)^3$.

B.2 If $x+y+z=0$ then find the value of $x^3+y^3+z^3$.

B.3 Evaluate :

(i) $(5x + 4y)^2$ **(ii)** $(4x - 5y)^2$ **(iii)** $\left(2x - \frac{1}{x}\right)^2$

B.4 Without actually calculating the cubes ,evaluate the expression $30^3+(-18)^3+(-12)^3$.

B.5 If $x = \sqrt{7} - \sqrt{5}$,$y = \sqrt{5} - \sqrt{3}$,$z = \sqrt{3} - \sqrt{7}$, then find the value of $x^3 + y^3 + z^3 - 2xyz$.

B.6 If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.

B.7 If $x + y = 3$ and $xy = -18$, find the value of $x^3 + y^3$.

B.8 If $a^4 + \frac{1}{a^4} = 119$, then find the value of $a^3 - \frac{1}{a^3}$.

B.9 Evaluate : $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$.

B.10 Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ is always non - negative for all values of a, b & c.

B.11 Prove that : $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

B.12 If a + b + c = 15, $a^2+b^2+c^2 = 83$, then find the value of $a^3+b^3+c^3-3abc$.

B.13 Find the value of $x^3-8y^3-36xy-216$ when x = 2y + 6.

Section (C) : Factorization

C.1 Factorize :

(i) $25x^2-10x+1-36y^2$ **(ii)** $2x^2+3\sqrt{5}\,x+5$

(iii) $\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$ **(iv)** $2y^3+y^2-2y-1$

C.2 Factorize :

(i) $(x+1)(x+2)(x+3)(x+4)-3$ **(ii)** $64a^3-27b^3-144a^2b+108ab^2$

(iii) $x^4+2x^3y-2xy^3-y^4$ **(iv)** $x^4+x^3-7x^2-x+6$

(v) $x^3-23x^2+142x-120$ **(vi)** $x^3+13x^2+32x+20$

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction and classification of polynomials

A.1 If $x^{51}+51$ is divided by (x + 1) the remainder is :
(A) 0 (B) 1 (C) 49 (D) 50

A.2 $\sqrt{2}$ is a polynomial of degree :
(A) 2 (B) 0 (C) 1 (D) $\frac{1}{2}$

A.3 The remainder obtained when the polynomial p(x) is divided by (b – ax) is :
(A) $p\left(\frac{-b}{a}\right)$ (B) $p\left(\frac{a}{b}\right)$ (C) $p\left(\frac{b}{a}\right)$ (D) $p\left(\frac{-a}{b}\right)$

A.4 The coefficient of x^2 in $(3x^2-5)(4+4x^2)$ is :
(A) 12 (B) 5 (C) – 8 (D) 8

A.5 Which of the following is a quadratic polynomial in one variable ?
(A) $\sqrt{2x^3}+5$ (B) $2x^2+2x^{-2}$ (C) x^2 (D) $2x^2+y^2$

A.6 If p (x) = $2+\frac{x}{2}+x^2-\frac{x^3}{3}$, then p (–1) is :
(A) $\frac{15}{6}$ (B) $\frac{17}{6}$ (C) $\frac{1}{6}$ (D) $\frac{13}{6}$

A.7 If (x + a) is a factor of $x^2 + px + q$ and $x^2 + mx + n$ then the value of a is :

(A) $\frac{m-p}{n-q}$ (B) $\frac{n-q}{m-p}$ (C) $\frac{n+q}{m+p}$ (D) $\frac{m+p}{n+q}$

A.8 If $x^2 - 4$ is a factor of $2x^3 + ax^2 + bx + 12$, where a and b are constant. Then the values of a and b are :

(A) – 3, 8 (B) 3, 8 (C) –3, – 8 (D) 3, – 8

A.9 The value of p for which x + p is a factor of $x^2 + px + 3 - p$ is :

(A) 1 (B) – 1 (C) 3 (D) – 3

A.10 Which of the following is cubic polynomial.

(A) $x^3 + 3x^2 - 4x + 3$ (B) $x^2 + 4x - 7$ (C) $3x^2 + 4$ (D) $3(x^2 + x + 1)$

Section (B) : Algebraic identity

B.1 The product of (x + a) (x + b) is :

(A) $x^2 + (a + b)x + ab$ (B) $x^2 - (a - b)x + ab$ (C) $x^2 + (a - b)x + ab$ (D) $x^2 + (a - b)x - ab$.

B.2 The value of 150 × 98 is :

(A) 10047 (B) 14800 (C) 14700 (D) 10470

B.3 The expansion of $(x + y - z)^2$ is :

(A) $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ (B) $x^2 + y^2 - z^2 - 2xy + yz + 2zx$

(C) $x^2 + y^2 + z^2 + 2xy - 2yz - 2zx$ (D) $x^2 + y^2 - z^2 + 2xy - 2yz - 2zx$

B.4 The value of $(x + 2y + 2z)^2 + (x - 2y - 2z)^2$ is :

(A) $2x^2 + 8y^2 + 8z^2$ (B) $2x^2 + 8y^2 + 8z^2 + 8xyz$

(C) $2x^2 + 8y^2 + 8z^2 - 8yz$ (D) $2x^2 + 8y^2 + 8z^2 + 16yz$

B.5 The value of $25x^2 + 16y^2 + 40xy$ at x = 1 and y = – 1 is :

(A) 81 (B) – 49 (C) 1 (D) None of these

B.6 On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get :

(A) $8a^2$ (B) $8a^2b$ (C) $8a^3b$ (D) $8a^3$

B.7 Find the value of $\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$, when a = – 5, b = – 6, c = 10.

(A) 1 (B) –1 (C) 2 (D) –2

B.8 If (x + y + z) = 1, xy + yz + zx = –1, xyz = –1, then value of $x^3 + y^3 + z^3$ is :

(A) –1 (B) 1 (C) 2 (D) –2

B.9 If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$ then which one of the following expression is correct :

(A) $x^3 + y^3 + z^3 = 0$ (B) $x + y + z = 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

(C) x + y + z = 3xyz (D) $x^3 + y^3 + z^3 = 3xyz$

Section (C) : Factorization

C.1 Factors of $(a + b)^3 - (a - b)^3$ is :

(A) $2ab(3a^2 + b^2)$ (B) $ab(3a^2 + b^2)$ (C) $2b(3a^2 + b^2)$ (D) $3a^2 + b^2$

C.2 Factors of $(42 - x - x^2)$ are :

(A) (x – 7)(x – 6) (B) (x + 7)(x – 6) (C) (x + 7)(6 – x) (D) (x + 7)(x + 6)

C.3 Factors of $\left(x^2 + \frac{x}{6} - \frac{1}{6}\right)$ are :

(A) $\frac{1}{6}$ (2x + 1)(3x + 1) (B) $\frac{1}{6}$ (2x + 1)(3x – 1) (C) $\frac{1}{6}$ (2x – 1)(3x – 1) (D) $\frac{1}{6}$ (2x – 1)(3x + 1)

C.4 Factors of polynomial $x^3 - 3x^2 - 10x + 24$ are :

(A) (x – 2)(x + 3)(x – 4) (B) (x + 2)(x + 3)(x + 4)
(C) (x + 2)(x – 3)(x – 4) (D) (x – 2)(x – 3)(x – 4)

C.5 One of the factors of the expression $(2a + 5b)^3 + (2a - 5b)^3$ would be :

(A) 4a (B) 10b (C) 2a + 5b (D) 2a – 5b

C.6 One of the factors of $(x-1) - (x^2 - 1)$ is:

(A) $x^2 - 1$ (B) x +1 (C) x – 1 (D) x + 4

Exercise-2

OBJECTIVE QUESTIONS

1. If $x + \frac{1}{x} = 5$, the value of $\frac{x^4+1}{x^2}$ is :

(A) 21 (B) 23 (C) 25 (D) 30

2. Which two of the following can be factorised with integral coefficients ?

I. $x^4 + x^2 + 1$
II. $x^4 + 2x + 2$
III. $x^4 - 2x^2 + 1$
IV. $x^4 = x + 1$

(A) I and II (B) I and IV (C) II and III (D) I and III

3. A factor of $x^3 - 6x^2 - 6x + 1$, is :

(A) x + 1 (B) x – 1 (C) x – 2 (D) 2x + 1

4. Let $x = (2008)^{1004} + (2008)^{-1004}$ and $y = (2008)^{1004} - (2008)^{-1004}$ then the value of $(x^2 - y^2)$ is equal to :

(A) 4 (B) – 4 (C) 0 (D) None

5. If $x^2 + \frac{1}{x^2} = 62$, then the value of $x^4 + \frac{1}{x^4}$ is :

(A) $8^4 - 2^8 - 2$ (B) $8^4 + 2$ (C) $8^4 - 2^8 + 2$ (D) $8^4 + 2^8 - 2$

6. If a + b + c = 0 then value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$ is :

(A) 1 (B) – 1 (C) 0 (D) 3

7. If x + y = – 4, then the value of $x^3 + y^3 - 12xy + 64$ will be

(A) 0 (B) 128 (C) 64 (D) – 64

8. The value of $\left[\frac{a^2 - 5ab}{a^2 - 6ab + 5b^2} \times \frac{a^2 - b^2}{a^2 + ab}\right]$ is :

(A) – 1 (B) $\frac{a}{b}$ (C) $\frac{1}{a}$ (D) 1

9. Evaluate : $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$.

(A) 0 (B) 1 (C) 2 (D) 3

10. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ then $\frac{a}{b} + \frac{b}{a} =$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{5}{6}$ (D) $\frac{6}{5}$

11. $\frac{x^{-3} - y^{-3}}{x^{-3}y^{-1} + (xy)^{-2} + y^{-3}x^{-1}} =$

(A) $x + y$ (B) $y - x$ (C) $\frac{1}{x} - \frac{1}{y}$ (D) $\frac{1}{x} + \frac{1}{y}$

12. If $\frac{\left(\sqrt{a} - \sqrt{b}\right)^2 + 4\sqrt{ab}}{a-b} = \frac{5}{3}$, then the value of a : b is :

(A) 1 : 16 (B) 1 : 4 (C) 4 : 1 (D) 16 : 1

13. If the polynomial $P(x) = 2x^4 + x^3 - 5x^2 - x + 1$ is divided by the polynomial $Q(x) = x^2 - x$ then the remainder is a linear polynomial $R(x) = ax + b$. Then $(a + b)$ equals :

(A) – 2 (B) –1 (C) 1 (D) 2

14. The polynomial $P(x) = x^4 + 4x^3 + 5x + 8$ is :

(A) divisible by (x + 2) but not divisible by (x + 1)
(B) divisible by (x + 1) as well as (x + 2)
(C) divisible by (x + 1) but not divisible by (x + 2)
(D) neither divisible by (x + 1) nor by (x + 2)

15. The value of k for which x + k is a factor of $x^3 + kx^2 - 2x + k + 4$ is :

(A) – 5 (B) 2 (C) $-\frac{4}{3}$ (D) $\frac{6}{7}$

16. If a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5, then (a + b + c + d) is equal to :

(A) – 5 (B) – 10/3 (C) – 7/3 (D) 5/3

17. If $x = \sqrt{2+\sqrt{2}}$, then $x^4 + \frac{4}{x^4}$ is :

(A) $2(3 - \sqrt{2})$ (B) 6 – 2 (C) $6 - \sqrt{2}$ (D) 12

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. One of the factors of the expression $x^4 + 8x$ is: **[Raj. NTSE Stage-1 2006]**

(A) $x^2 + 2$ (B) $x^2 + 8$ (C) x + 2 (D) x – 2

2. One of the factors of the expression $(2x - 3y)^2 - 7(2x - 3y) - 30$ is : **[Raj. NTSE Stage-1 2007]**

(A) 2x – 3y – 10 (B) 2x – 3y + 10 (C) 3x – 2y + 5 (D) 6x – 4y – 15

3. If $x + \frac{1}{x} = 3$, then the value of $x^6 + \frac{1}{x^6}$ is : **[Raj. NTSE Stage-1 2013]**

(A) 927 (B) 114 (C) 364 (D) 322

4. If $(a-5)^2 + (b-c)^2 + (c-d)^2 + (b+c+d-9)^2 = 0$, then the value of $(a+b+c)(b+c+d)$ is : **[Harayana NTSE Stage-1 2013]**
(A) 0 (B) 11 (C) 33 (D) 99

5. If $x + y + z = 1$, then $1-3x^2-3y^2-3z^2+2x^3+2y^3+2z^3$ is equal to : **[Harayana NTSE Stage-1 2013]**
(A) 6xyz (B) 3xyz (C) 2xyz (D) xyz

6. If $x + \frac{1}{x} = 4$, then the value of $x^6 + \frac{1}{x^6}$ is : **[Delhi NTSE Stage - 1 2013]**
(A) 927 (B) 114 (C) 364 (D) 2702

7. If $a + b = 6$ and $ab = 8$, then $a^3 + b^3 =$ .... **[Gujarat NTSE Stage - 1 2013]**
(A) 18 (B) 36 (C) 54 (D) 72

8. If polynomial $P(x) = 3x^3 - x^2 - ax - 45$ has one zero of 3, then a = ... **[Gujarat NTSE Stage - 1 2013]**
(A) 3 (B) 6 (C) 9 (D) 12

9. If one factor of $27x^3 + 64y^3$ is $(3x + 4y)$ what is the second factor ? **[Gujarat NTSE Stage - 1 2013]**
(A) $(3x^2 - 4y)$ (B) $(3x^2 + 12xy + 4y^2)$ (C) $(9x^2 + 12xy - 16y^2)$ (D) $(9x^2 - 12xy + 16y^2)$

10. Which one of the following is a factor of the expression $(a+b)^3 - (a-b)^3$? **[Madhya Pradesh NTSE Stage-1 2013]**
(A) a (B) $3a^2 - b$ (C) 2b (D) $(a+b)(a-b)$

11. If x + 3 divides $x^3 + 5x^2 + kx$, then k is equal to : **[Odisha NTSE Stage-1 2013]**
(A) 2 (B) 4 (C) 6 (D) 8

12. If $x^2 - x - 1 = 0$, then the value of $x^3 - 2x + 1$ is **[Harayana NTSE Stage-1 2014]**
(A) 0 (B) 2 (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{1-\sqrt{5}}{2}$

13. If x% of y is equal to 1% of z, y% of z is equal to 1% of x and z% of x is equal to 1% of y, then the value of xy + yz + zx is - **[Harayana NTSE Stage-1 2014]**
(A) 1 (B) 2 (C) 3 (D) 4

14. If $(x+a)^2 + (y+b)^2 = 4(ax+by)$, where x, a, y, b are real, the value of xy – ab is : **[West Bengal NTSE Stage-1 2014]**
(A) a (B) 0 (C) b (D) None of these

15. If $a + b + c = 0$ and $a^2 + b^2 + c^2 = k(a^2 - bc)$ then k = ... **[Bihar NTSE Stage-1 2014]**
(A) 0 (B) 1 (C) 2 (D) 3

16. If (x – 2) is a factor of polynomial $x^3 + 2x^2 - kx + 10$. Then the value of k will be : **[Chattisgarh NTSE Stage-1 2014]**
(A) 10 (B) 13 (C) 16 (D) 9

17. If $\frac{x+a}{b+c} + \frac{x+b}{c+a} + \frac{x+c}{a+b} + 3 = 0$, $a > 0$, $b > 0$, $c > 0$, then the value of x is : **[Delhi NTSE Stage - 1 2014]**
(A) $-(a^2 + b^2 + c^2)$ (B) $(a + b + c)$ (C) $-(a + b + c)$ (D) $\sqrt{a+b+c}$

18. If $x = \frac{1}{1+\sqrt{2}}$, then value of $x^2 + 2x + 3$ is : **[Delhi NTSE Stage - 1 2014]**

(A) 3 (B) 0 (C) 4 (D) 1

19. If $x + \frac{1}{x} = 5$, then $x^3 - 5x^2 + x + \frac{1}{x^3} - \frac{5}{x^2} + \frac{1}{x} =$ .... : **[Bihar NTSE Stage-1 2014]**

(A) –5 (B) 0 (C) 5 (D) 10

20. If $x + y = 1$ then $x^3 + y^3 + 3xy =$ ..... **[Jharkhand NTSE Stage - 1 2014]**

(A) 0 (B) 1 (C) 2 (D) None of these

21. If $x - y = 5$, $xy = 24$ then the value of $x^3 + y^3$ will be - **[Uttar Pradesh NTSE Stage-1 2014]**

(A) 23 (B) 73 (C) 65 (D) 74

22. If $x + \frac{1}{x} = 2$ then $\sqrt{x} + \frac{1}{\sqrt{x}}$ will be – **[Uttar Pradesh NTSE Stage-1 2014]**

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{2} + 1$ (D) 1

23. If $x + y = 8$, $xy = 15$, the value of $x^2 + y^2$ will be **[Uttar Pradesh NTSE Stage-1 2014]**

(A) 32 (B) 34 (C) 36 (D) 38

24. If $p - q = -8$ and p.q. = – 12 then the value of $p^3 - q^3$ is : **[Madhya Pradesh NTSE Stage-1 2014]**

(A) 224 (B) – 224 (C) 242 (D) – 242

25. $(a + b + c)(ab + bc + ca) - abc$ is equal to the **[Madhya Pradesh NTSE Stage-1 2014]**

(A) $(a + b)(c + b)(c + a)$ (B) $(a - b)(b + c)(c + a)$

(C) $(a + b)(b - c)(c + a)$ (D) $(a + b)(b + c)(c - a)$

26. Find the factors of the polynomial $8a^3 + 27b^3 + 64c^3 - 72abc$. **[Maharashtra NTSE Stage-1 2014]**

(A) $(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab + 12bc - 8ac)$

(B) $(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 + 6ab - 12bc + 8ac)$

(C) $(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 8ac)$

(D) $(2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab - 12bc + 8ac)$

27. If $\frac{p}{q} + \frac{q}{p} = 2$, what is the value of $\left(\frac{p}{q}\right)^{23} + \left(\frac{q}{p}\right)^{7}$ **[Delhi NTSE Stage - 1 2015]**

(A) 0 (B) 2 (C) –2 (D) none of these

28. Value of $x\left[\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)-1\right]$ is **[Delhi NTSE Stage - 1 2015]**

(A) 3 (B) 2x (C) 5x (D) 1

29. Simplify the value of $\frac{3.75 \times 3.75 + 1.25 \times 1.25 - 2 \times 3.75 \times 1.25}{3.75 \times 3.75 - 1.25 \times 1.25}$ **[Delhi NTSE Stage - 1 2015]**

(A) 5.0 (B) 0.5 (C) 2.5 (D) 1.5

30. If $p(x) = 2x^3 - 3x^2 + 5x - 4$ is divided by $(x - 2)$, what is remainder ? **[Gujarat NTSE Stage - 1 2015]**

(A) 12 (B) 8 (C) 10 (D) –10

31. What is the co-efficient of x^2y^2 in the expansion of $(x + y)^4$? **[Gujarat NTSE Stage - 1 2015]**

(A) 3 (B) 4 (C) 5 (D) 6

32. Zeroes of which quadratic polynomial are 4 and 3. **[Gujarat NTSE Stage - 1 2015]**
(A) $x^2 + 7x + 12$ (B) $x^2 - 7x + 12$ (C) $x^2 + 7x - 12$ (D) $x^2 - 7x - 12$

33. If $x^2 - 3x + 1 = 0$, then the value of $x^5 + \frac{1}{x^5}$ **[Jharkhand NTSE Stage - 1 2015]**
(A) 87 (B) 123 (C) 135 (D) 201

34. If $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$ and $\frac{yz}{y+z} = c$, where a, b, c are non-zero numbers, then the value of x ?
[Jharkhand NTSE Stage - 1 2015]
(A) $\frac{2abc}{ab+ac-bc}$ (B) $\frac{2abc}{ac+bc-ab}$ (C) $\frac{abc}{ab+bc+ac}$ (D) $\frac{2abc}{ab+bc-ac}$

35. If pqr = 1, then the value of $\left(\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}\right)$ **[Odisha NTSE Stage-1 2015]**
(A) 0 (B) pq (C) 1 (D) pq

36. The square root of $x^{b^2}x^{b^2+2ab}x^{a^2-b^2}$ is **[Rajasthan NTSE Stage-1 2016]**
(A) $x^{2(a+b)}$ (B) $x^{\frac{a+b}{2}}$ (C) $x^{\frac{(a+b)^2}{2}}$ (D) X^{a+b}

37. If a + b + c = 0, then the value of $\frac{(a+b)^2}{ab}+\frac{(b+c)^2}{bc}+\frac{(c+a)^2}{ca}$ is **[Rajasthan NTSE Stage-1 2016]**
(A) 1 (B) 2 (C) 3 (D) – 3

38. One of the factors of $81a^4 + (x - 2a)(x - 5a)(x - 8a)(x - 11a)$ is **[Haryana NTSE Stage-1 2016]**
(A) $x^2 - 13ax + 31a^2$ (B) $x^2 + 13ax + 31a^2$ (C) $x^2 + 18ax - 31a^2$ (D) $x^2 - 18ax + 31a^2$

39. If $f\left(2x+\frac{1}{x}\right) = x^2 + \frac{1}{4x^2} + 1 (x \neq 0)$, the value of f(x) is **[West Bengal NTSE Stage-1 2016]**
(A) $4x^2$ (B) $\frac{1}{4}\left(2x+\frac{1}{x}\right)^2$ (C) $\frac{1}{4}x^2$ (D) $4\left(2x+\frac{1}{x}\right)^2$

40. If $2r = h + \sqrt{r^2 + h^2}$, the value of r : h is $(r, h \neq 0)$ **[West Bengal NTSE Stage-1 2016]**
(A) 4 : 3 (B) 3 : 4 (C) 1 : 2 (D) 2 : 1

41. Let a, b, x, y be real numbers such that $a^2 + b^2 = 81$, $x^2 + y^2 = 121$ and $ax + by = 99$. Then the values of $ay - bx$ is : **[West Bengal NTSE Stage-1 2016]**
(A) –1 (B) 1 (C) 0 (D) None of these

42. The value of $\frac{(0.03)^2-(0.01)^2}{0.03-0.01}$ is **[Bihar NTSE Stage-1 2016]**
(A) 0.02 (B) 0.004 (C) 0.4 (D) 0.04

43. If (x+ 2), is a factor of $2x^3 - 5x + k$, then the value of k is **[Raj. NTSE Stage-1 2016]**
(A) 6 (B) – 6 (C) 26 (D) -26

44. If a + b + c = 0, then the value of $\frac{(a+b)^2}{ab}+\frac{(b+c)^2}{bc}+\frac{(c+a)^2}{ca}$ is **[Raj. NTSE Stage-1 2016]**
(A) 1 (B) 2 (C) 3 (D) -3

45. The simplified form of the expression given below is **[Delhi NTSE Stage - 1 2016]**

$$\frac{\frac{y^4 - x^4}{x(x+y)} - \frac{y^3}{x}}{y^2 - xy + x^2}$$

(A) 1 (B) 0 (C) –1 (D) 2

46. If $a = \frac{4xy}{x+y}$, the value of $\frac{a+2x}{a-2x} + \frac{a+2y}{a-2y}$ in most simplified form is **[Delhi NTSE Stage - 1 2016]**

(A) 0 (B) 1 (C) –1 (D) 2

47. If x, y, z are real numbers such that $\sqrt{x-1} + \sqrt{y-2} + \sqrt{z-3} = 0$ then the values of x, y, z are respectively **[Delhi NTSE Stage - 1 2016]**

(A) 1, 2, 3 (B) 0, 0, 0 (C) 2, 3, 1 (D) 2, 4, 1

48. If x – 2 is a factor of $3x^4 - 2x^3 + 7x^2 - 21x + k$, then the value of k is
[Gujarat NTSE Stage - 1 2016]

(A) 2 (B) 9 (C) 18 (D) – 18

49. If 2x +3y + z = 0 then $8x^3 + 27y^3 + z^3 \div xyz$ is equal to
[Uttar Pradesh NTSE Stage-1 2017]

(A) 0 (B) 6 (C) 18 (D) 9

50. If $p = x + \frac{1}{x}$ then the value of $p - \frac{1}{p}$ will be- **[Uttar Pradesh NTSE Stage-1 2017]**

(A) 3x (B) $\frac{3}{x}$ (C) $\frac{x^4 + x^2 + 1}{x^3 + x}$ (D) $\frac{x^4 + 3x^2 + 1}{x^3 + x}$

51. Factors of $\frac{1}{3}c^2 - 2c - 9$ are- **[Uttar Pradesh NTSE Stage-1 2017]**

(A) $\left(\frac{1}{3}c + 3\right)(c + 3)$ (B) $\left(\frac{1}{3}c - 3\right)(c - 3)$ (C) $\left(\frac{1}{3}c - 3\right)(c + 3)$ (D) $\left(c - \frac{1}{3}\right)(3c + 1)$

Answer Key

BOARD LEVEL EXERCISE

TYPE (I)

1. 0 **2.** Not defined **3.** 1 **4.** 5 **5.** 50

6. 27 **7.** ¼ **8.** 3abc **9.** a = 2

10. **(i)** (x + 3) (x + 6) **(ii)** (3x – 1) (2x + 3) **(iii)** (2x + 3) (x – 5) **(iv)** 2 (6 – r) (r + 7)

11. **(i)** 1092727 **(ii)** 10302 **(iii)** 998001

TYPE (II)

12. **(i)** NO **(ii)** YES **13.** 1/3

14. **(i)** 77760 **(ii)** 3 (x – y) (y – z) (z – x)

15. **(i)** 6 **(ii)** 1/5 **(iii)** – 1 **(iv)** 1/5

16. **(i)** 10x **(ii)** $x^{20} + 1$ **(iii)** $2x^2 - x - 1$

17. $\frac{-31}{4}$ **18.** $Q = x^3 + x^2 + x + 1$; R = 2 **20.** m = – 2

TYPE (III)

21. **(i)** $(a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ac)$
(ii) $(\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac)$

22. **(i)** 0 **(ii)** 0 **23.** 756 **24.** – 1

25. $x^3 - 8y^3 - z^3 - 6xyz$ **27.** $-250y^3 - 120x^2y$

TYPE (IV)

29. 62

EXERCISE - 1

SUBJECTIVE QUESTIONS

Section (A)

A.1 0 **A.2** – 8 **A.3** infinite **A.4** – 3 **A.5** yes

A.7 $\frac{-13}{3}$ **A.8** p = 3, q = – 3

Section (B)

B.1 997002999 **B.2** 3xyz

B.3 **(i)** $25x^2 + 16y^2 + 40xy$ **(ii)** $16x^2 + 25y^2 - 40xy$ **(iii)** $4x^2 + \frac{1}{x^2} - 4$.

B.4 19440 **B.5** $-4\sqrt{5} + 2\sqrt{3} + 2\sqrt{7}$ **B.6** 370. **B.7** 189.

B.8 – 36. **B.9** 3. **B.12** 180. **B.13** 0

Section (C)

C.1 **(i)** (5x – 1 – 6y) (5x – 1 + 6y). **(ii)** $(x + \sqrt{5})(2x + \sqrt{5})$.

(iii) $\left(x + \frac{1}{x} - 2\right)^2$ **(iv)** (2y + 1) (y + 1)(y – 1)

C.2 **(i)** $(x^2 + 5x + 3)(x^2 + 5x + 7)$. **(ii)** $(4a - 3b)^3$.
(iii) $(x + y)^3 (x - y)$. **(iv)** (x – 1) (x + 1) (x + 3) (x – 2).
(v) (x – 12) (x – 10) and (x – 1) **(vi)** (x +1)(x + 2)(x + 10).

OBJECTIVE QUESTIONS

Section (A)

A.1 (D) **A.2** (B) **A.3** (C) **A.4** (C) **A.5** (C)
A.6 (B) **A.7** (B) **A.8** (C) **A.9** (C) **A.10** (A)

Section (B)

B.1 (A) **B.2** (C) **B.3** (C) **B.4** (D) **B.5** (C)
B.6 (D) **B.7** (A) **B.8** (B) **B.9** (B)

Section (C)

C.1 (C) **C.2** (C) **C.3** (B) **C.4** (A) **C.5** (A)
C.6 (C)

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	A	C	D	A	D	D	A	B	D	A	C	C
Ques.	16	17													
Ans.	B	D													

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	D	D	A	D	D	C	D	C	C	B	C	B	C
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	C	B	B	B	B	B	B	A	C	B	A	B	C
Ques.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	D	B	B	B	C	C	C	C	C	A	C	D	A	C	C
Ques.	46	47	48	49	50	51									
Ans.	D	A	D	C	C	C									