## MATHEMATICS

# **Class-IX**

# **Topic-15 CONSTRUCTIONS**



INDEX					
S. No.	Торіс	Page No.			
1.	Theory	1 – 12			
2.	Exercise (Board Level)	13			
3.	Exercise-1	14 – 15			
4.	Answer Key	15			



## CH-15 CONSTRUCTIONS

### A. CONSTRUCTIONS

#### (a) To construct the bisector of a line segment

#### STEPS :

- (i) Draw a line segment AB of given length.
- (ii) With centre A and radius more than half of AB, draw arcs, one on each side of AB.
- (iii) With B as centre and the same radius as before, draw arcs, cutting the previously drawn arcs at E and F respectively.
- (iv) Join EF intersecting AB at M. Then M bisects the line segment AB as shown in figure.



**Justification :** Let us see how the above steps of construction give us the perpendicular bisector of AB. Join A and B both to E and F to form EA, EB, FA and FB.

In triangles EAF and EBF, we have

AE = BE [:: Arcs of equal radii are equal]

AF = BF	[∵ Arcs of equal radii are equal]

EF = EF [Common]

So, by SSS - criterion of congruence, we have

```
\Delta EAF \cong \Delta EBF
```

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∠AEM = ∠BEM
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In triangles EMA and EMB, we have

EA = EB [:: Arcs of equal radii are equal]

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EM = EM [Common]
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∠AEM = ∠BEM	[From (	i)
		י.

```
So, by SAS congruence criterion, we have
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- $\Delta EMA \cong \Delta EMB$
- $\Rightarrow$  AM = BM and  $\angle$ EMA =  $\angle$ EMB
- But,  $\angle EMA$  and  $\angle EMB$  form a linear pair.
- ∴ ∠EMA + ∠EMB = 180°
- $\Rightarrow$   $\angle EMA = \angle EMB = 90^{\circ}$  [ $\because \angle EMA = \angle EMB$ ]
- Thus, we have

AM = BM and  $\angle$ EMA =  $\angle$ EMB = 90°

Hence, EF is the perpendicular bisector of AB.

Following examples will illustrate the above procedure.

#### (b) To construct the bisector of a given angle

Let ABC be the given angle to be bisected.







#### STEPS :

- (i) With B as centre and a suitable radius, draw an arc which cuts ray BA at point D and ray BC at point E.
- (ii) Taking D and E as centres and with equal radii draw arcs which intersect each other at point F. In this step, each equal radius must be more than half the length DE.
- (iii) Join B and F and produce to get the ray BF.

Ray BF is the required bisector of the given angle ABC. Justification : Join DF and EF.



In  ${\scriptscriptstyle\Delta}\,\text{BDF}$  and  ${\scriptscriptstyle\Delta}\,\text{BEF}$  :

	BD = BE	[Radii of the same arc]	
	DF = EF	[Radii of the equal arcs]	
	BF = BF	[Common]	
$\Rightarrow$	$\Delta \operatorname{BDF} \cong \Delta \operatorname{BEF}$	[By SSS]	
$\Rightarrow$	$\angle$ DBF = $\angle$ EBF	[By cpctc]	
i.e.,	$\angle ABF = \angle CBF$	$\Rightarrow$ BF bisects $\angle$ ABC.	Hence Proved.

#### (c) To construct the required angle

(i) To Construct the Required Angle of 60° :



#### STEPS :

(I) Draw a line BC of any suitable length.

(II) With B as centre and any suitable radius, draw an arc which cuts BC at point D.

(III) With D as centre and radius same, as taken in step (II), draw one more arc which cuts previous arc at point E.

(IV) Join BE and produce upto any point A. Then, ∠ ABC = 60°

#### (ii) To Construct an Angle of 120° :



#### STEPS :

(I) Draw a line BC of any suitable length.

(II) Taking B as centre and with any suitable radius, draw an arc which cuts BC at point D.





(III) Taking D as centre, draw an arc of the same radius, as taken in step (II), which cuts the first arc at point E.

(IV) Taking E as centre and radius same, as taken in step (II), draw one more arc which cuts the first arc at point F.

(V) Join BF and produce upto any suitable point A. Then,  $\angle$  **ABC = 120°** 

(iii) To Construct an Angle of 30° :



#### STEPS :

(I) Construct angle ABC = 60° by compass.

(II) Draw BD, the bisector of angle ABC. Then,  $\angle$  DBC = 30°

#### (iv) To Construct an Angle of 90° :



#### STEPS :

(I) Construct angle ABC = 120° by using compass.

(II) Draw PB, the bisector of angle EBG. Then, ∠ PBC = 90°

#### Alternative Method :



- (I) Draw a line segment BC of any suitable length.
- (II) Produce CB upto an arbitrary point O.
- (III) Taking B as centre, draw an arc which cuts OC at points D and E.

(IV) Taking D and E as centres and with equal radii draw arcs

which cut each other at point P. [The radii in this step must be of length more than half of DE.]

(V) Join BP and produce. Then, ∠ PBC = 90°

#### (v) To Construct an Angle of 45° :







#### STEPS :

(I) Draw  $\angle$  PBC = 90° (II) Draw AB which bisects angle PBC. Then,  $\angle$  **ABC = 45**°

#### **Alternative Method :**



#### STEPS :

- (I) Construct  $\angle$  ABC = 60°.
- (II) Draw BD, the bisector of angle ABC.
- (III) Draw BE, the bisector of angle ABD.
- Then, ∠ EBC = 45°
- (vi) To Construct an Angle of 105° :



#### STEPS :

(I) Construct  $\angle$  ABC = 120° and  $\angle$  PBC = 90°

(II) Draw BO, the bisector of  $\angle$  ABP. Then,  $\angle$  **OBC = 105°** 

#### (vii) To Construct an Angle of 150°.



#### STEPS :

(I) Draw line segment BC of any suitable length. Produce CB upto any point O.

(II) With B as centre, draw an arc (with any suitable radius) which cuts OC at points D and E.

(III) With D as centre, draw an arc of the same radius, as taken in step (II), which cuts the first arc at point F.

(IV) With F as centre, draw one more arc of the same radius, as taken in step (II), which cuts the first arc at point G.

(V) Draw PB, the bisector of angle EBG.

Now  $\angle$  FBD =  $\angle$  GBF =  $\angle$  EBG = 60° Then,  $\angle$  **PBC = 150**°





#### (viii) To Construct an Angle of 135°.



#### STEPS :

(I) Construct  $\angle$  PBC = 150° and  $\angle$  GBC = 120°

(II) Construct BQ, the bisector of angle PBG. Then,  $\angle$  **QBC = 135°** 

#### (d) To construct a Triangle

#### (i) To construct an equilateral triangle when its one side is given.

In order to construct an equilateral triangle when the measure (length) of its side is given, we follow the following steps :

#### STEPS :

(I) Draw a ray AX with initial point A.



(II) With centre A and radius equal to length of a side of the triangle draw an arc BY, cutting the ray AX at B.

(III) With centre B and the same radius draw an arc cutting the arc BY at C.

(IV) Join AC and BC to obtain the required triangle.

(ii) When the base of the triangle, one base angle and the sum of other two sides are given.

#### STEPS :

(I) Obtain the base, base angle and the sum of other two sides. Let AB be the base,  $\angle A$  be the base angle and  $\ell$  be the sum of the lengths of other two side BC and CA of  $\triangle ABC$ .



(II) Draw the base AB.

(III) Draw  $\angle$ BAX of measure equal to that of  $\angle$ A.

(IV) From ray AX, cut off line segment AD equal to  $\ell$  (the sum of other two dies).

(V) Join BD

(VI) Draw the perpendicular bisector of BD meeting AD at C.

(VII) Join BC to obtain the required triangle ABC.

Justification : Let us now see how do we get the required triangle.

Since point C lies on the perpendicular bisector of BD. Therefore,

CD = CB

Now, AC = AD - CD  $\Rightarrow AC = AD - BC [:: CD = CB]$  $\Rightarrow AD = AC + CB$ 





(iii) When the base of the triangle, one base angle and the difference of the other two sides are given.



#### STEPS :

(I) Obtain the base, base angle and the difference of two other sides. Let AB be the base,  $\angle A$  be the base angle and I be the difference of the other two sides BC and CA of  $\triangle ABC$ . i.e., I = AC - BC, if AC > BC or, I = BC - AC, if BC > AC

(II) Draw the base AB of given length.

(III) Draw  $\angle$ BAX of measure equal to that of  $\angle$ A.

(IV) If AC > BC, then cut off segment AD = AC – BC from ray AX. [in figure (i)]. If AC < BC, then extend XA to X'; on opposite side of AB and cut off segment AD = BC – AC from ray AX' [in figure (ii)].

(V) Join BD.

(VI) Draw the perpendicular bisector of BD which cuts AX or AX', as the case may be, at C.

(VII) Join BC to obtain the required triangle ABC.

**Justification :** Let us now see how do we get the required triangle. Since C lies on the perpendicular bisector of DB.

So, AD = AC - CD = AC - BC.

(iv) When the perimeter of the triangle and both the base angles are given.



#### STEPS :

(I) Obtain the perimeter and the base angles of the triangle. Let ABC be a triangle of perimeter p cm and base BC.

(II) Draw a line segment XY equal to the perimeter p of  $\triangle ABC$ .

- (III) Construct  $\angle$ YDX and  $\angle$ XYE.
- (IV) Draw bisectors of angles  $\angle$ YXD and  $\angle$ XYE and mark their intersection point as A.
- (V) Draw the perpendicular bisectors of XA and YA meeting XY in B and C respectively.
- (VI) Join AB and AC to obtain the required triangle ABC.





Justification : For the justification of the construction, we observe that B lies on the perpendicular bisector of AX. XB = AB $\angle AXB = \angle BAX$ *.*..  $\Rightarrow$ Similarly, C lies on the perpendicular bisector of AY. YC = AC∠AYC = ∠YAC ÷  $\Rightarrow$ XY = XB + BC + CYXY = AB + BC + ACNow.  $\Rightarrow$ In  $\triangle AXB$ , we have  $\angle ABC = \angle AXB + \angle BAX = 2 \angle AXB = \angle BXD = \angle B$ 

## **Solved Examples**

#### Example.1

In ∆AYC,

Draw a line segment of length 7.8 cm, draw the perpendicular bisector of this line segment.

**Sol.** Let the given line segment be AB = 7.8 cm. **STEPS :** 

 $\angle ACB = \angle AYC + \angle YAC = 2 \angle AYC = \angle CYE = \angle C$ .



(i) Draw the line segment AB = 7.8 cm.

(ii) With point A as centre and a suitable radius, more than half the length of AB, draw arcs on both the sides of AB.

(iii) With point B as centre and with the same radius draw arcs on both the sides of AB. Let these arc cut at points P & Q as shown in the figure.

(iv) Draw a line through the points P and Q. The line so obtained is the required perpendicular bisector of given line segment AB.

Line PQ is perpendicular bisector of AB.

- (A) PQ bisects AB i.e., OA = OB.
- (B) PQ is perpendicular to AB
- i.e.,  $\angle POA = \angle POB = 90^{\circ}$ .







Now, in  $\triangle$  APO &  $\triangle$  BPO AP = BP[By construction] OP = OP[Common side]  $\angle APO = \angle BPO$ [Proved above]  $\Delta \text{ APO} \cong \Delta \text{ BPO}$ [By SAS]  $\Rightarrow$ OA = OB  $\Rightarrow$  $\angle POA = \angle POB = \frac{180^{\circ}}{2} = 90^{\circ} [\because \angle POA + \angle POB = 180^{\circ}]$ And,  $\Rightarrow$ PQ is perpendicular bisector of AB.

#### Example. 2

Draw an equilateral triangle having each side of 2.5 cm.

Sol. Given one side of the equilateral triangle be 2.5 cm.



#### STEPS :

(i) Draw a line segment BC = 2.5 cm.

(ii) Through B, construct ray BP making angle 60° with BC.

i.e., ∠ PBC = 60°

(iii) Through C, construct CQ making angle 60° with BC

i.e., ∠ QCB = 60°

(iv) Let BP and CQ intersect each other at point A.

#### Then, $\Delta$ **ABC** is the required equilateral triangle.

**Proof :** Since,  $\angle ABC = \angle ACB = 60^{\circ}$ 

- $\therefore$   $\angle$  BAC = 180° (60° + 60°) = 60°
- $\Rightarrow$  All the angles of the  $\Delta$  ABC drawn are equal.
- $\Rightarrow$  All the sides of the  $\triangle$  ABC drawn are equal.
- $\Rightarrow$   $\Delta$  ABC is the required equilateral triangle.

Hence Proved.

#### Alternative method :

If one side is 2.5 cm, then each side of the required equilateral triangle is 2.5 cm.



#### STEPS :

- (i) Draw BC = 2.5 cm
- (ii) With B as centre, draw an arc of radius 2.5 cm
- (iii) With C as centre, draw an arc of radius 2.5 cm
- (iv) Let the two arcs intersect each other at point A. Join AB and AC.





#### Example. 3

Construct a triangle with 3 cm base and sum of other two sides is 8 cm and one base angle is 60°. Given the base BC of the triangle ABC be 3 cm, one base angle  $\angle$  B = 60° and the sum of the other Sol.



#### STEPS :

- (i) Draw BC = 3 cm
- (ii) At point B, draw PB so that ∠PBC = 60°
- (iii) From BP, cut BD = 8 cm.
- (iv) Join D and C.
- (v) Draw perpendicular bisector of CD, which meets BD at point A.
- (ví) Join Á and C.

Thus,  $\triangle ABC$  is the required triangle.



Proof : Since, OA is perpendicular bisector of CD

$$\Rightarrow$$
 OC = OD

Also, OA = OA [Common]

- $\triangle AOC \cong \triangle AOD$ *.*..
- AC = AD $\Rightarrow$

BD = BA + AD = BA + AC = Given sum of the other two sides *.*..

[By SAS]

Thus, base BC and  $\angle B$  are drawn as given and BD = BA + AC.

Hence Proved.

Then,  $\triangle ABC$  is the required equilateral triangle.

#### Example, 4

Construct a right triangle, when one side is 3.8 cm and the sum of the other side and hypotenuse is 6 cm.

Sol. Here, if we consider the required triangle to be  $\Delta$  ABC, as shown alongside. Clearly, AB = 3.8 cm,  $\angle$  B = 90° and BC + AC = 6 cm.







#### STEPS :

- (i) Draw AB = 3.8 cm
- (ii) Through B, draw line BP so that  $\angle$  ABP = 90°
- (iii) From BP, cut BD = 6 cm
- (iv) Join A and D.
- (v) Draw perpendicular bisector of AD, which meets BD at point C.

Thus, **∆ABC** is the required triangle.



#### Example. 5

Construct a triangle with base of 8 cm and difference between the length of other two sides is 3 cm and one base angle is 60°.

**Sol.** Given the base BC of the required triangle ABC be 8 cm i.e., BC = 8 cm, base angle B = 60° and the difference between the lengths of other two sides AB and AC be 3 cm.

i.e., AB – AC = 3 cm or AC – AB = 3 cm. (I) When AB – AC = 3 cm i.e., AB > AC :



#### STEPS :

- (i) Draw BC = 8 cm
- (ii) Through point B, draw BP so that  $\angle$  PBC = 60°.
- (iii) From BP cut BD = 3 cm.
- (iv) Join D and C.
- (v) Draw perpendicular bisector of DC ; which meets BP at point A.
- (vi) Join A and C.

#### Thus, $\Delta$ ABC is the required triangle.



Proof :Since, OA is perpendicular bisector of CD

$\Rightarrow$	OD = OC	
	$\angle$ AOD = $\angle$ AOC = 90°	
And,	OA = OA	[Common]
<i>:</i> .	$\Delta \text{ AOD} \cong \Delta \text{ AOC}$	[By SAS]
$\Rightarrow$	AD = AC	[By cpctc]





Now, BD = BA - AD = BA - AC [:: AD = AC]

= Given difference of other two sides.

Thus, the base BC and  $\angle$  B are drawn as given and BD = BA – AC.

Hence Proved.

(II) When AC - AB = 3 cm i.e., AB < AC :



#### STEPS :

(i) Draw BC = 8 cm

(ii) Through B, draw line BP so that angle PBC = 60°.

- (iii) Produce BP backward upto a suitable point Q.
- (iv) From BQ, cut BD = 3 cm.
- (v) Join D and C.
- (vi) Draw perpendicular bisector of DC, which meets BP at point A.
- (vii) Join A and C.

Thus,  $\Delta$  **ABC** is the required triangle.



Proof : Since, OA is perpendicular bisector of CD

FIGH. Since, OA is perpendicular disector of CD						
$\Rightarrow$	OD = OC					
	$\angle$ AOD = $\angle$ AOC = 90°					
And,	OA = OA	[Common]				
<i>.</i> :.	$\Delta \text{ AOD} \cong \Delta \text{ AOC}$	[By SAS]				
$\Rightarrow$	AD = AC	[By cpctc]				
Now	BD = AD - AB = AC - AB	$[\cdot \cdot AD = AC]$				

= Given difference of other two sides.

Thus, the base BC and  $\angle$  B are drawn as given and BD = AC – AB. Hence Proved.

#### Example. 6

Construct a triangle ABC with AB + BC + CA = 12 cm,  $\angle$  B = 45° and  $\angle$  C = 60°.

**Sol.** Given the perimeter of the triangle ABC be 12 cm i.e., AB + BC + CA = 12 cm and both the base angles be 45° and 60° i.e.,  $\angle$  B = 45° and  $\angle$  C = 60°.

#### STEPS :

- (i) Draw a line segment PQ = 12 cm
- (ii) At P, construct line PR so that  $\angle$  RPQ = 45° and at Q, construct a line QS so that  $\angle$  SQP = 60°.
- (iii) Draw bisector of angles RPQ and SQP which meet each other at point A.
- (iv) Draw perpendicular bisector of AP, which meets PQ at point B.
- (v) Draw perpendicular bisector of AQ, which meets PQ at point C.
- (vi) Join AB and AC.







#### **Check Your Level**

- **1.** Draw a line AB of length 8 cm divide it into two equal parts.
- 2. Construct the angles of the following measurement.

(a) 
$$30^{\circ}$$
 (b)  $22\frac{1}{2}$  (c)  $15^{\circ}$ 

- 3. Construct a triangle ABC with the following data BC = 4.7 cm,  $\angle$  B = 43°, AB + AC = 9.2 cm.
- 4. Construct a triangle ABC with the following data  $\angle B = 43^\circ$ ,  $\angle C = 37^\circ$ , perimeter 6.8 cm.
- 5. Construct a triangle ABC with the following data BC = 6 cm, AC AB = 2 cm,  $\angle B = 60^{\circ}$ .



[01 MARK EACH]

## **Exercise Board Level**

#### TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS :

**1.** Find the difference of BC and AC for which construction of a triangle ABC in which AB = 4 cm,  $\angle A = 60^{\circ}$  is not possible.

#### (Q.No. 2 to 7) : Write True or False in each of the following. Give reasons for your answer:

- 2. An angle of 52.5° can be constructed.
- **3.** An angle of 42.5° can be constructed.
- 4. A triangle ABC can be constructed in which AB = 5 cm,  $\angle A = 45^{\circ}$  and BC + AC = 5 cm.
- 5. A triangle ABC can be constructed in which BC = 6 cm,  $\angle$  C = 30° and AC AB = 4 cm.
- 6. A triangle ABC can be constructed in which  $\angle B = 105^\circ$ ,  $\angle C = 90^\circ$  and AB + BC + AC = 10 cm.
- 7. A triangle ABC can be constructed in which  $\angle B = 60^\circ$ ,  $\angle C = 45^\circ$  and AB + BC + AC = 12 cm.

#### TYPE (II) : SHORT ANSWER TYPE QUESTIONS :

- 8. Draw an angle of 110° with the help of a protractor and bisect it. Measure each angle.
- **9.** Draw a line segment AB of 4 cm in length. Draw a line perpendicular to AB through A and B, respectively. Are these lines parallel ?
- Draw an angle of 80° with the help of a protractor. Then construct angles of (i) 40° (ii) 160° and (iii) 120°.
- **11.** Construct a triangle whose sides are 3.6 cm, 3.0 cm and 4.8 cm. Bisect the smallest angle and measure each part.
- **12.** Construct a triangle ABC in which BC = 5 cm,  $\angle$  B = 60° and AC + AB = 7.5 cm.
- **13.** Construct a square of side 3 cm.
- **14.** Construct a rectangle whose adjacent sides are of lengths 5 cm and 3.5 cm.
- **15.** Construct a rhombus whose side is of length 3.4 cm and one of its angles is 45°.

#### TYPE (III) : LONG ANSWER TYPE QUESTIONS:

#### (Q.No. 16 to 20) Construct each of the following and give justification :

- **16.** A triangle if its perimeter is 10.4 cm and two angles are 45° and 120°.
- **17.** A triangle PQR given that QR = 3cm,  $\angle$  PQR = 45° and QP PR = 2 cm.
- **18.** A right triangle when one side is 3.5 cm and sum of other sides and the hypotenuse is 5.5 cm.
- **19.** An equilateral triangle if its altitude is 3.2 cm.
- **20.** A rhombus whose diagonals are 4 cm and 6 cm in lengths.





#### [02 MARKS EACH]

#### [03 MARK EACH]



## **Exercise-1**

#### SUBJECTIVE QUESTIONS

#### Subjective Easy, only learning value problems

#### Section (A) : Constructions

- **A-1.** For each angle, given below, make a separate construction. Draw a ray BC and an another ray BA so that the ∠ ABC is equal to :
  - (i)  $15^{\circ}$  (ii)  $22\frac{1^{\circ}}{2}$  (iii)  $75^{\circ}$  (iv)  $52\frac{1^{\circ}}{2}$ (v)  $67\frac{1^{\circ}}{2}$  (vi)  $165^{\circ}$  (vii)  $135^{\circ}$
- A-2. Construct an equilateral triangle with side :
  (i) 5 cm (ii) 5.4 cm (iii) 6.2 cm
- A-3. Construct a triangle ABC, in which :
  - (i) base AB = 5.4 cm,  $\angle$  B = 45° and AC + BC = 9 cm.
  - (ii) base BC = 6 cm,  $\angle$  B = 60° and AB + AC = 9.6 cm.
  - (iii) base AC = 5 cm,  $\angle$  C = 90° and AB + BC = 10.6 cm.
- A-4. Construct a right triangle, with base = 4 cm and the sum of the other side and hypotenuse = 9.4 cm.
- A-5. Construct a triangle ABC, in which :
  - (i) BC = 4.8 cm,  $\angle B = 45^{\circ} \text{ and } AB AC = <math>2.4 \text{ cm}$ .
  - (ii) BC = 4.8 cm,  $\angle B = 45^{\circ} \text{ and } AC AB = 2.4 \text{ cm}$ .
  - (iii) AB = 5.3 cm,  $\angle A = 60^{\circ}$  and AC BC = 2 cm.
  - (iv)  $AB = 5.3 \text{ cm}, \angle A = 60^{\circ} \text{ and } BC AC = 2 \text{ cm}.$
- A-6. Construct a triangle ABC, with :
  - (i) perimeter = 12 cm,  $\angle B = 45^{\circ}$  and  $\angle C = 60^{\circ}$ .
  - (ii) perimeter = 11.6 cm,  $\angle B = 60^{\circ}$  and  $\angle C = 90^{\circ}$ .
  - (iii) perimeter = 11 cm,  $\angle A = 60^{\circ}$  and  $\angle C = 45^{\circ}$ .
  - (iv) perimeter = 10 cm,  $\angle B = \angle C = 60^{\circ}$ .
- **A-7.** Without finding the length of each side of the equilateral triangle construct it. If its perimeter is 16 cm.
- **A-8.** Construct a  $\triangle$  PQR in which base QR = 4 cm,  $\angle$  R = 30° and PR PQ = 1.1 cm.

#### **OBJECTIVE QUESTIONS**

#### Single Choice Objective, straight concept/formula oriented

#### Section (A) : Constructions

A-1. With the help of a ruler and compass, it is possible of construct an angle of : (A)  $37^{\circ}$  (B)  $40^{\circ}$  (C)  $37.5^{\circ}$  (D)  $48.5^{\circ}$ 







Answer Key

## **Exercise-1**

#### **OBJECTIVE QUESTIONS**

Section (A)									
A-1.	(C)	A-2.	(B)	A-3.	(D)	A-4.	(A)	A-5.	(C)

